

Volatility as an asset

Equity volatility is an asset class

We believe equity index volatility meets the definition of an asset class:

- (1) selling index volatility offers significant, passively generated returns produced by facilitating hedging flow for equity and credit investors;
- (2) returns are large enough to justify a nontrivial allocation; and
- (3) volatility selling tends to outperform long equities in hostile markets, offering an appealing diversification benefit.

High VRP drives outsized risk-adjusted returns

Volatility Risk Premiums (VRPs) have been very high over time, resulting in short S&P 500 variance strategies having Sharpe ratios 4X US equities and outperforming 12 of 13 CS/Tremont hedge fund indices on a risk-adjusted basis. Returns were so strong that even modest short volatility allocations in a portfolio resulted in significant expansion of the efficient frontier. We back-test over 50 systematic S&P 500 volatility strategies and rank-order performance and Sharpe ratios.

Introducing REPs: A tool for sizing, risk analysis

We develop a new method for sizing and tracking the performance of volatility positions, which allows us to rank “best-in-class” volatility strategies across listed and OTC alternatives. Risk Equivalent Portfolios (REPs) represent a breakthrough in volatility strategy analysis, allowing investors to size a volatility position to meet specific risk targets.

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Executive summary: Equity volatility as an asset class

We believe equity index volatility meets the definition of an asset class and should be considered by investors within an asset allocation framework. Equity index volatility selling strategies have generated consistently high returns, with high Sharpe ratios, often substantially outperforming major equity indices and hedge fund strategies, even at extremely modest levels of allocation. They have also tended to outperform long equity strategies in hostile markets.

Despite this attractive profile, we believe volatility selling strategies have been underutilized by investors due to a lack of performance data, little or no information on sizing, and a related lack of focus on risk management in a portfolio context. We address these issues and provide a framework to guide investors through strategy selection and proper sizing. While we focus on S&P 500 index volatility strategies in this report, our arguments hold true for other indices that are the subject of large investments and resulting hedging demand.

Equity index volatility meets the definition of an asset

What defines a new asset class? In our view, at a minimum, there is the expectation that a passive position in that asset will produce significant returns above cash over time. Specifically, an asset should have:

- long-run returns that are not dependent upon the skill of the investor or manager,
- returns of sufficient magnitude to justify a nontrivial allocation, and
- significant diversification benefits in hostile markets.

We show that equity index volatility meets all these criteria. First, selling index volatility offers significant returns that are produced by facilitating hedging flow for both equity and credit portfolios. These returns do not depend on the skill of the manager; as we will show, passively selling various types of index volatility offers significant returns. Due to its highly attractive risk/return profile, even small allocations in short volatility strategies can have a significant impact on returns. Finally, volatility strategies tend to outperform long equity portfolios in hostile markets, offering an appealing diversification benefit.

Sizing is key: Introducing REPs, a new method for calibrating risk

We develop a new method for sizing and tracking the performance of volatility positions that allows us to rank “best-in-class” volatility strategies across listed and OTC alternatives. Risk Equivalent Portfolios (REPs) represent a breakthrough in volatility strategy analysis, allowing investors to size a volatility position to meet specific risk targets and directly compare strategies across terms and notionals. We provide a sizing methodology to fit a continuum of risk budgets, rather than a “one-size-fits-all” solution.

REPs contain one risk-free asset (LIBOR) and one risky asset (a short variance swap), where the amount of variance sold is determined by our sizing equations such that the aggregate portfolio is calibrated to meet investor risk/return objectives.

For brevity, we focus on three S&P 500 short variance swap REPs: (1) LoVol REP (5% volatility per annum), designed to have similar or lower risk than many hedge fund alternatives, (2) MedVol REP (10% vol per annum), similar in risk to ATM option selling strategies such as the BXM, and (3) HiVol REP (15% vol), similar in risk to the S&P 500.

1. A substantial Volatility Risk Premium leads to strong passively generated returns

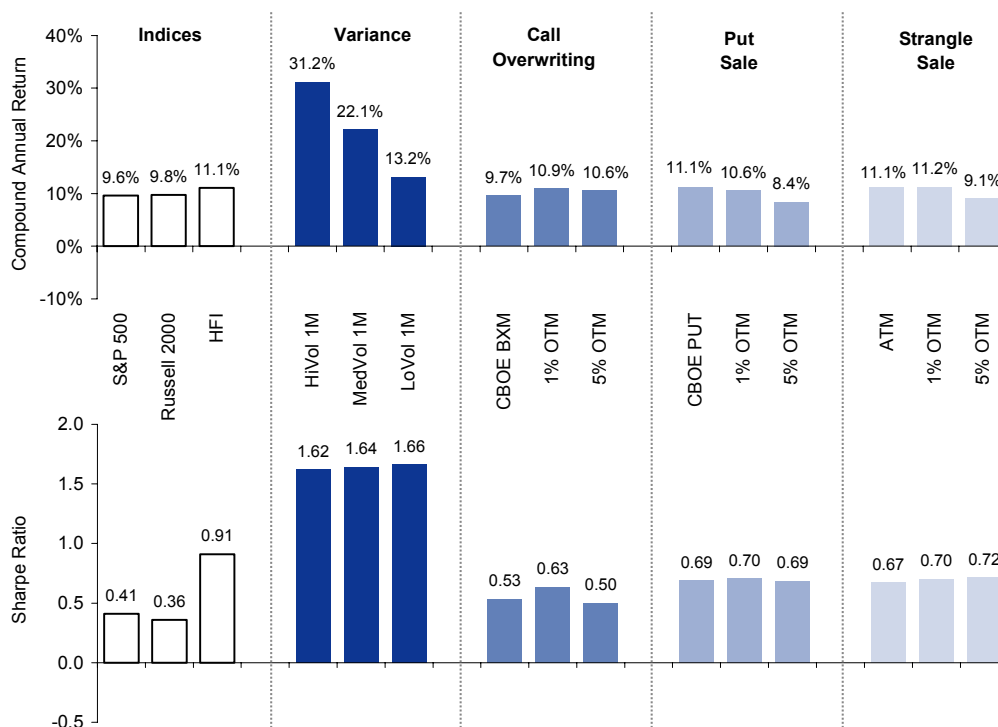
We find a substantial Volatility Risk Premium embedded in the S&P 500 options market. From January 1996 to September 2007, the average spread between S&P 500 ATM implied and subsequent realized volatility was 2.3 vol points across the months in our study, with a 10-point spread for deep OTM put options. We found strong risk-adjusted returns were obtained from a variety of option selling strategies due in part to the high volatility risk premium. We analyze the returns from a wide range of S&P 500 volatility selling strategies including, call overwriting, naked put and strangle selling, short variance swaps, and short forward variance strategies. We include the returns from the CBOE S&P 500 BuyWrite (BXM) and PutWrite Index (PUT) for independent comparison.

The rank order of strategy returns and Sharpe ratios follows a clear pattern in terms of the type of volatility exposure offered, with the highest returns accruing to strategies that directly capture the implied – realized Volatility Risk Premium via variance swaps.

- S&P 500 short one-month variance swap strategies, designed to be risk-equivalent to the S&P 500 in terms of the standard deviation of monthly returns, had monthly success rates of 81%, over 3 times the annual return of the S&P 500 (31.2% vs. 9.6%), and Sharpe ratios of 1.6. The strategy outperformed the BXM by 21.5% per annum and generated over 2.5 times the return from selling ATM puts or straddles. Rolling one-year returns never dropped below zero over the course of our study.
- S&P 500 variance strategies had Sharpe ratios 4 times higher than US equities and outperformed 12 of the 13 CS/Tremont hedge fund indices on a risk-adjusted basis.

Exhibit 1: S&P 500 short variance strategies had 3 times the annual return of the S&P 500 with similar risk. Strong risk-adjusted returns were obtained from a variety of vanilla option selling strategies due in part to the high Volatility Risk Premium.

One-month option selling strategies. Sharpe ratios based on monthly observations (1996–2007)



Note: HFI refers to Credit Suisse/Tremont Hedge Fund Index

Source: CBOE, Goldman Sachs Research estimates.

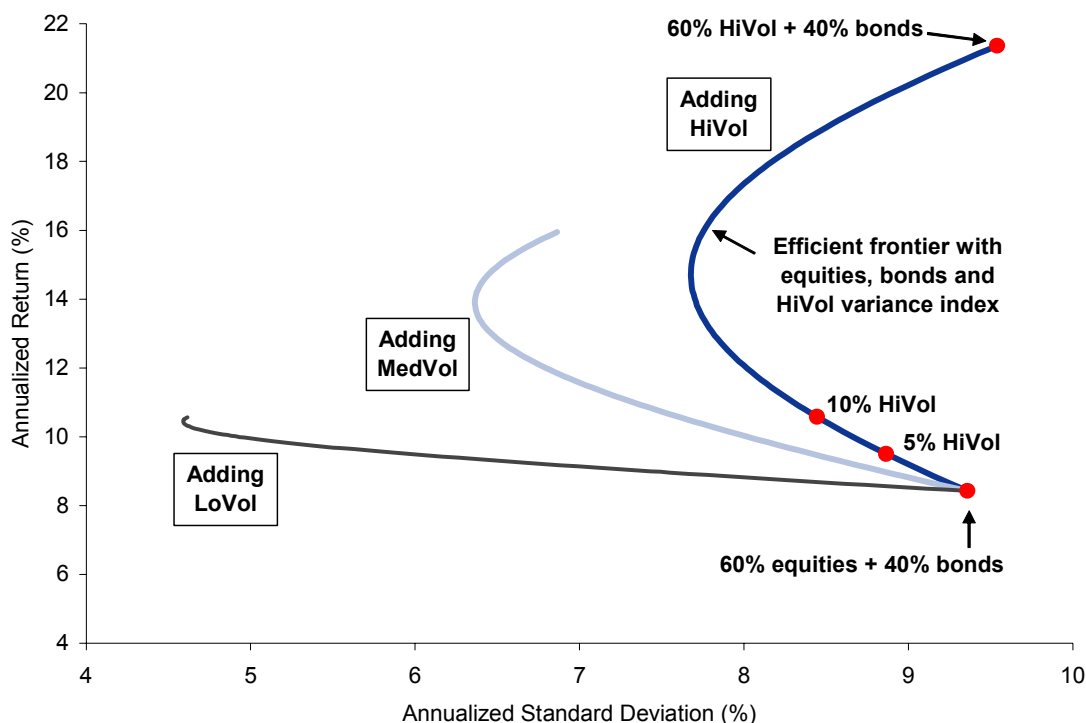
2. Volatility in an asset allocation framework: Strong returns lead to significant expansion of the efficient frontier

Given the strong risk-adjusted returns, it is not surprising that adding a short S&P 500 variance position to a traditional 60/40 equity bond portfolio leads to a significant expansion of the efficient frontier. Defining our equity exposure to be S&P 500 total returns and using the Ibbotson Long-Term Government Bond total return index as our bond proxy, we found:

- Replacing large-cap equity with a 5% investment in the HiVol variance REP outperformed a benchmark 60/40 equity-bond portfolio by 108 basis points per annum with a 50-bp reduction in risk. Sharpe ratios increased from 0.44 to 0.58.
- Across the 10 largest calendar-month declines in the S&P 500 from January 31, 1996 to September 30, 2007, the new portfolio outperformed the 60/40 equity-bond benchmark in 9 out of 10 months, with an average outperformance of 30 bp per month.

Exhibit 2: Adding SPX variance to a 60/40 equity-bond portfolio leads to significant expansion of the efficient frontier
compound annual returns; annualized volatility of monthly returns, January 31, 1996–September 30, 2007

Portfolio Weights			w/ HiVol Variance Index			w/ MedVol Variance Index			w/ LoVol Variance Index		
Equity	Bond	Variance Index	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio
60%	40%	0%	8.42	9.36	0.44	8.42	9.36	0.44	8.42	9.36	0.44
59%	40%	1%	8.64	9.26	0.47	8.55	9.24	0.46	8.46	9.23	0.45
58%	40%	2%	8.86	9.15	0.49	8.68	9.13	0.48	8.50	9.11	0.46
57%	40%	3%	9.07	9.06	0.52	8.80	9.02	0.50	8.53	8.98	0.47
56%	40%	4%	9.29	8.96	0.55	8.93	8.91	0.52	8.57	8.86	0.48
55%	40%	5%	9.50	8.87	0.58	9.05	8.80	0.54	8.60	8.74	0.49
⋮											
50%	40%	10%	10.58	8.44	0.74	9.68	8.28	0.65	8.78	8.13	0.55



Source: Ibbotson, Goldman Sachs Research estimates.

3. High risk premium leads to outperformance in hostile markets

We dedicate a large portion of this paper to strategy risk. One appealing characteristic of S&P 500 call overwriting strategies is that they tend to outperform in bear markets, with the call premium providing a downside performance cushion. On average, we find even stronger hostile market results from short variance, put, and strangle selling strategies.

In fact, during the worst bear market in recent history (9/00-9/02), variance selling strategies calibrated to be risk-equivalent to the SPX were up 45% vs. a 45% decline in the SPX and achieved that performance with a volatility 4 points lower than the market, 14% versus 18%.

For option-based trades, bear market strategy performance has historically been in line with the amount of premium sold, with wider breakevens for straddles leading to better performance in down markets. The argument is different for variance strategies, which are less dependent on market direction than on the path of the index (large moves versus a slow drift). **On average, we found S&P 500 variance swaps priced in a 3.3 vol point or 30% monthly increase in realized market volatility. The large risk premium provided a strong positive performance cushion across a wide range of potential index paths (in both up and down markets).**

In general, short variance strategies experience negative returns under a substantial re-pricing of risk. De-levering variance returns using our REP methodology helped reduced risk and minimize drawdowns.

Capturing the risk premium: All strategies are not created equal

Although a large volatility risk premium benefits many types of volatility selling strategies, we found large differences in returns across strategy types. In terms of risk-adjusted performance: S&P 500 variance outperformed vanilla options selling, which outperformed forward variance strategies. The primary return drivers are different for each of these strategies, with each offering a different exposure to volatility:

Variance swaps are the purest play on the volatility risk premium embedded in option prices and offer distinct advantages over traditional option strategies for capturing the implied versus realized spread: (1) the payoff is directly linked to the difference between implied and realized variance; (2) variance swaps require no delta hedge; and (3) variance strikes trade at a premium to ATM implied vol allowing for higher profit potential on the short.

Non-delta-hedged vanilla option strategies tend to be more highly correlated to directional market moves than a pure play on volatility (delta first, vol second). Our past research indicates that the success of vanilla option-based strategies can hinge as much on minimizing exercise cost as on capturing a rich option premium.

Forward variance trades are designed to express views on the direction of implied volatility. The most well-known example is the exchange-listed VIX future, a market-based expectation for future VIX levels. **Forward variance trades do not capture the implied versus realized volatility risk premium.** In our 2006 report "VIX futures over the last decade," we found long forward variance trades had (1) high payouts in periods of market stress, (2) high negative betas to the market; and (3) lower carry costs on long positions than many other long vol strategies—one reason we often recommend forward variance to express tactical views on volatility. Because forward variance trades capture a different exposure, which does not tend to trade at a substantial premium, we find forward variance trades often provide the best way to "get long" volatility. The increased liquidity in exchange-listed VIX options and VIX futures has made the VIX market a popular and transparent way of implementing tactical volatility trades.

Why does the volatility risk premium exist?

The ability to hedge allows investment managers to externalize short-term equity market risk and focus on investing for the long term. As such, hedges are a useful tool, but are not free. In order to attain downside protection, investment managers must pay a premium for put hedges—one that is large enough to attract liquidity providers and/or investors to provide that protection. This cycle of supply and demand creates the Volatility Risk Premium (VRP) and drives the long-run returns for short index options strategies.

We are often asked why the VRP has not been “arbed” away. Historically, the supply from arbitrageurs has been small relative to hedging demand and there are limits to arbitrage. We believe investors are needed to fill in the gap. Even as investors become more involved in this market, as we show, only a small amount of short volatility exposure is needed to generate sizable returns within acceptable risk levels. This means that short volatility investors are unlikely to outsize the hedging market, in our view.

Hedges provide real value to investors, reducing portfolio volatility

Hedging allows investment managers to reduce short-term equity risk and lower portfolio volatility. The ability to externalize short-term price risk allows investors to better utilize existing capital, either by: (1) retaining assets that should outperform over the long run but may experience short-term dislocations from fundamentals or (2) investing more aggressively. Investors ultimately benefit from the reduced portfolio volatility.

Supply/demand: Investors tend to be net buyers of index volatility

Evidence of a supply/demand imbalance in S&P 500 options stemming from put hedges is apparent in flow data. In their 2004 paper, “Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?”, Bollen and Whaley found that a majority of S&P 500 index options trading involves puts. In fact, 55% of all index options trades over the time period they analyzed were in puts, vs. only 33% for single stock options. What’s more, using tick data, they found that put activity is dominated by buyers, with the largest buying pressure in OTM and deep OTM puts. This confirms what we see in index skew data: the expensiveness of index put options is consistent with hedging supply/demand.

Hedgers buy at a premium to attract sufficient investor capital

Hedging transfers risk from equity portfolio managers to other investors or liquidity providers. In order to hedge, money managers must, therefore, attract sufficient capital into the derivatives market to bear this unwanted risk. This is accomplished by purchasing options at a premium to the market’s expected fair value.

Historically, broker-dealers have been the only real suppliers of hedges. The cost of managing the associated risk consists of: (1) the normal provision of capital for daily hedging costs, and (2) costs associated with managing the gap risk of a short option position, which cannot be entirely hedged away. Dealer risk profiles drive option prices higher, creating a premium in index option volatility. Bollen and Whaley note that “a market maker will not stand ready to sell an unlimited number of contracts in a particular options series. As his position grows large and imbalanced, his hedging costs and/or volatility-risk exposure also increase, and he is forced to charge a higher price.” **Outsized hedging demand relative to supply keeps index options trading at a volatility premium. This volatility risk premium generates the long-run returns required to attract sufficient capital into the derivatives market to supply liquidity.**

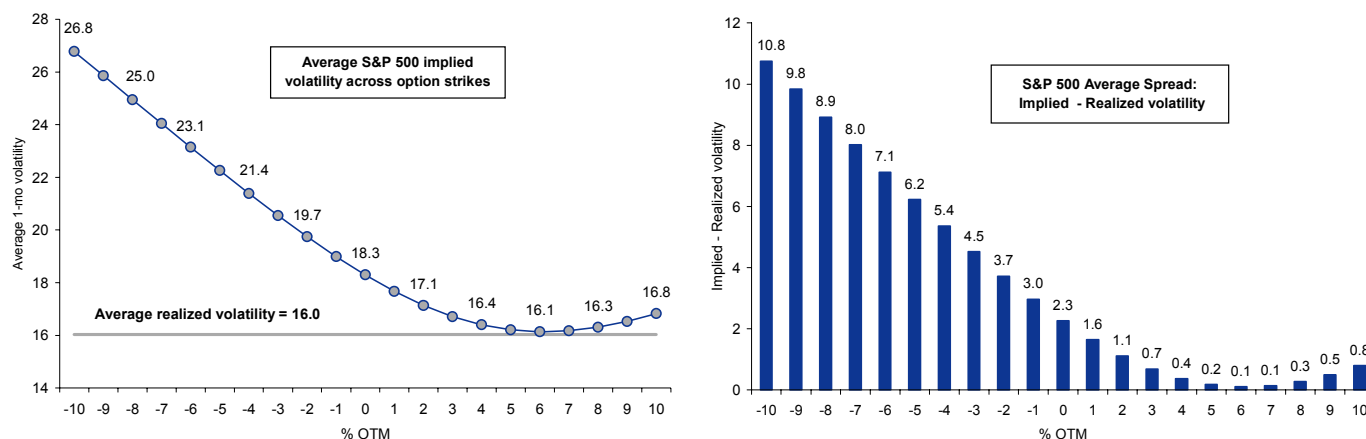
The volatility risk premium is well-documented in academic studies

A consistent VRP has been well-documented by academics and is confirmed by our analyses. The VRP has existed since listed index options began trading in 1983. As early as 1973, Fisher Black and Myron Scholes commented that “the actual prices at which options are bought and sold deviate in certain systematic ways from the values predicted by the formula. Option buyers pay prices that are consistently higher than those predicted by the formula....There are large transaction costs in the options market, all of which are effectively paid by options buyers.” Although printed more than 10 years before index options began trading in the listed market, the quote has remained true for more than 30 years.

More recently, Bondarenko (2003) studied the “overpriced puts puzzle” and concluded that put options on the S&P 500 are systematically overpriced, stating that “selling unhedged puts would have resulted extraordinary paper profits over the sample period.” He finds significant returns from put selling strategies and comments that “for ATM puts to break even (i.e., to have the average excess return of zero), crashes of the magnitude experienced in October 1987 would have to occur 1.3 times per year.” Similarly, Schneeweis and Spurgin (2001) and Bollen and Whaley (2004) found that implied index volatility traded rich relative to the volatility of realized market returns, generating strong returns from option selling strategies.

Bakshi and Kapadia (2003) find evidence of a volatility risk premium by examining delta-hedged S&P 500 option strategies, noting that “if option prices incorporate a nonzero volatility risk premium, then we can infer its existence from the returns of an option portfolio that has dynamically hedged all risks except volatility risk.” They find that after controlling for underlying S&P 500 index moves, call buyers pay the seller a premium of about \$0.43 per call and estimate the cumulative wealth transfer from buyers to sellers over the eight-year period analyzed (1988-1995) to be on the order of several billion dollars.

Exhibit 3: Average S&P 500 implied volatility across option strikes is well above subsequent realized volatility
monthly observations (January 1996–September 2007)



Source: Goldman Sachs Research estimates.

We find similar results in our performance data which we detail in the remainder of the paper. An analysis of the implied minus realized spread of index options over time clearly shows the VRP, as shown above in Exhibit 3:

Implied vs. realized volatility spread: The average spread between ATM implied and subsequent realized volatility was 2.3 vol points, with higher spreads for OTM puts. The difference for 5% OTM put options averaged 6.2 vol points and increased to 10.8 for 10% OTM puts. Calls sold 5%-8% OTM were essentially traded at fair volatility.

Skew: The pronounced implied volatility skew is apparent with the average implied volatility for 10% OTM puts at 26.8, about 46% higher than the implied vol for ATM options.

Investors are needed to fill in the gap; few have taken advantage

Why isn't the volatility premium "arbed away"? Few investors have stepped in to fill the gap between hedgers and liquidity providers, resulting in a high volatility risk premium to date. Specifically, we see the following factors contributing to the VRP:

1. Historically, broker-dealers have been the primary suppliers of hedges. As previously discussed, there are costs associated with taking on risks related to outsized hedging demand. More recently, professional arbitrageurs have been involved in selling index options to take advantage of the rich VRP. However, (1) the supply from arbitrageurs has been small relative to the aggregate hedging demand in our view; and (2) we show excess returns can be generated from selling even small amounts of volatility. In other words, there are few participants and the notional amount of volatility sold to generate high returns is low relative to hedge demand.
2. Academics cite practical limits to the effect arbitrageurs can have in bringing prices back to theoretical "fair value." Shleifer and Vishny (1997) argue that "arbitrage is conducted by a relatively small number of highly specialized investors who take large positions using other people's money." They argue that investor capital is often allocated based upon past fund performance and arbitrage returns tend to be low precisely when the opportunity gap widens and future returns are expected to be high. Therefore, arbitrageurs risk losing capital when mispricings are large, limiting their ability to close the fair value gap. **So arbitrageurs can lean against the mispricing, but their ability to do so is limited due to leverage constraints and little access to new capital. Investors are needed to fill the gap.**

The sustainability of long-run returns

Long-run returns for volatility strategies are therefore determined by the equilibrium between the market's desire to hedge and investors' willingness to bear the necessary risk. Thus for equity derivatives, as for any other asset, investor returns are determined by the demand and supply for risk capital. We believe the returns to short volatility strategies are likely to persist in the future and find that the implied vs. realized volatility spread has remained positive across volatility and market regimes. This will be true unless the community of investors willing to sell index volatility overwhelms the hedging demand. Given the size of the long-equity community and increased cross-over activity from credit investors, we believe that would be highly unlikely. In fact, as investors demand more transparency with stricter risk management controls, hedging with equity derivatives will arguably become an even more highly utilized tool in the investment process.

Volatility in an asset allocation framework: Expands efficient frontier

Although volatility strategies have generated impressive passive returns over time that seem of sufficient magnitude to justify a nontrivial allocation for investment managers, the next step is to examine how the asset would have performed in a portfolio context. In this section, we analyze how a volatility overlay would have affected the risk-return characteristics of an equity-bond portfolio as well as a simple long S&P 500 portfolio. Throughout this section, we define our equity exposure to be S&P 500 total returns and use the Ibbotson Long-Term Government Bond total return index as our bond proxy.

We focus on three primary questions to analyze the potential diversification benefits:

1. If we start with an equity-bond portfolio and perform a classic portfolio optimization, what allocation do we obtain for volatility?
2. What is the effect of adding small amounts of volatility to a standard 60/40 equity-bond portfolio?
3. How do modest volatility allocations affect the risk-return profile of an S&P 500 portfolio?

We analyze short S&P 500 one-month variance swap strategies in this section as they had the highest risk-adjusted returns across volatility strategies, but the same concepts can be applied to vanilla options strategies as well. Ibbotson Associates (2004) found that the BXM significantly expanded the efficient frontier when combined with conventional equity-bond portfolios. We found much stronger results from variance strategies with a substantial increase in Sharpe ratios from even modest allocations.

1. Volatility in a classic portfolio optimization: Significant expansion of efficient frontier

Our historical results indicate that investors have obtained over three times the annualized S&P 500 total return with “equity-like” risk for trades that directly capture the volatility risk premium through variance swaps. Given the strong risk-adjusted performance, it is not surprising that a classical portfolio optimization suggests a relatively high allocation to variance within a portfolio of equities and bonds.

In fact, based upon the returns on our HiVol S&P 500 one-month variance index and correlations from January 31, 1996, to September 30, 2007, a classic portfolio optimization would suggest that of the fraction of the portfolio not held in cash,

- 71% should be allocated to variance,
- 0% to equities, and
- 29% to government bonds.

The addition of volatility to the asset allocation mix leads to significant expansion of the efficient frontier, allowing the investor to achieve higher average returns for any given level of portfolio risk, as measured by portfolio volatility (Exhibit 14). As seen by the slope of the capital markets line, adding variance would have increased the portfolio Sharpe ratio from 0.46 to 1.82 over the years 1996-2007.

While we are clearly not advocating that investors should swap out of large portions of equity exposure into variance swap strategies, the framework does illustrate just how

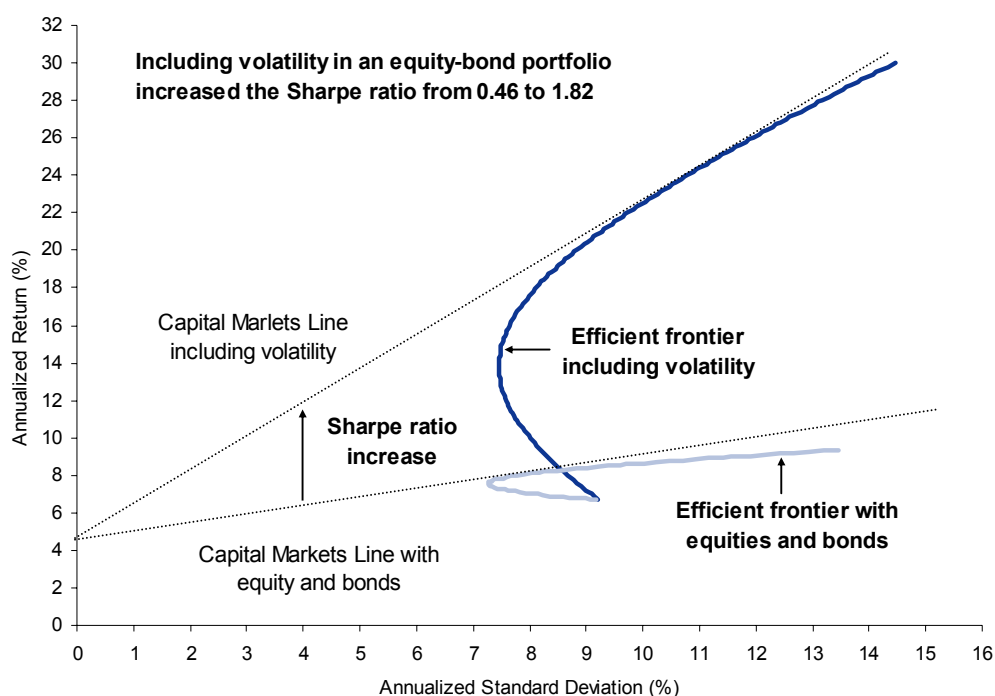
powerful a volatility overlay can be. We show in the next two sections that even a small allocation can have a powerful effect on portfolio performance.

Exhibit 13: Asset risk, returns and correlations (January 31, 1996–September 30, 2007)
annualized compound monthly returns; annualized volatility of monthly returns

	Annual Return	Annual Volatility	Correlation			
			S&P 500	Bond	Cash	HiVol
S&P 500	9.6%	14.8%	1.00	-0.15	0.01	0.48
Bond	6.7%	9.2%		1.00	-0.03	-0.11
Cash	4.3%	0.5%			1.00	-0.01
HiVol	31.2%	15.0%				1.00

Source: Ibbotson, Goldman Sachs Research estimates.

Exhibit 14: Including volatility in the asset allocation decision can lead to substantially improved risk-adjusted performance in a classic portfolio optimization
compound annual returns; annualized volatility of monthly returns



Source: Ibbotson, Goldman Sachs Research estimates.

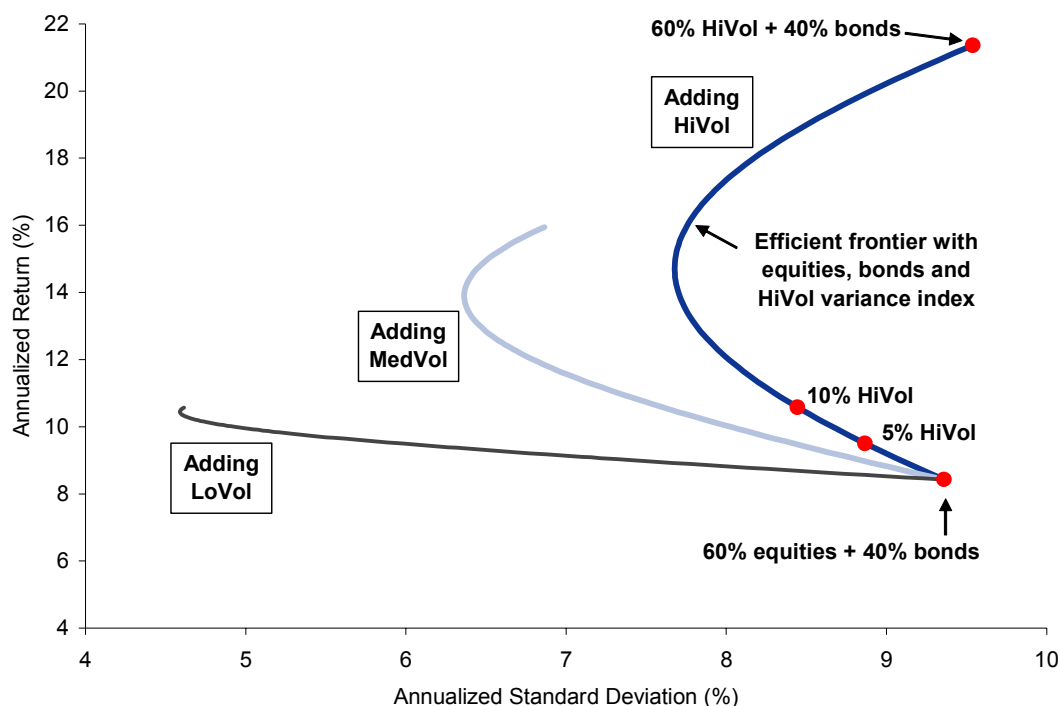
2. Combining volatility with a 60/40 equity-bond portfolio

Given that traditional investment managers are unlikely to replace large portions of their equity holdings with variance strategies, we show that even small allocations have a big enough impact on risk-adjusted returns to justify an investment. We examine the effect of replacing modest amounts of large-cap equity with variance in a 60/40 equity-bond portfolio:

- Replacing S&P 500 exposure with a 5% investment in the HiVol one-month variance REP outperformed the benchmark 60/40 portfolio by 108 bp per annum with a 49 bp reduction in risk. The portfolio had return of 9.50% per annum with a standard deviation of 8.87%.
- Increasing the allocation in the HiVol index to 10% led to a portfolio that outperformed the benchmark 60/40 portfolio by 216 bp per annum with a 92-bp reduction in risk. The resulting portfolio had an annual return of 10.58% with a standard deviation of 8.44% and increased the Sharpe ratio versus the benchmark portfolio from 0.44 to 0.74.

Exhibit 15: Adding SPX variance to a 60/40 equity-bond portfolio leads to significant expansion of the efficient frontier
compound annual returns; annualized volatility of monthly returns

Portfolio Weights			w/ HiVol Variance Index			w/ MedVol Variance Index			w/ LoVol Variance Index		
Equity	Bond	Variance Index	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio
60%	40%	0%	8.42	9.36	0.44	8.42	9.36	0.44	8.42	9.36	0.44
59%	40%	1%	8.64	9.26	0.47	8.55	9.24	0.46	8.46	9.23	0.45
58%	40%	2%	8.86	9.15	0.49	8.68	9.13	0.48	8.50	9.11	0.46
57%	40%	3%	9.07	9.06	0.52	8.80	9.02	0.50	8.53	8.98	0.47
56%	40%	4%	9.29	8.96	0.55	8.93	8.91	0.52	8.57	8.86	0.48
55%	40%	5%	9.50	8.87	0.58	9.05	8.80	0.54	8.60	8.74	0.49
⋮											
50%	40%	10%	10.58	8.44	0.74	9.68	8.28	0.65	8.78	8.13	0.55



Source: Ibbotson, Goldman Sachs Research estimates.

3. The benefits of combining S&P 500 and volatility portfolios

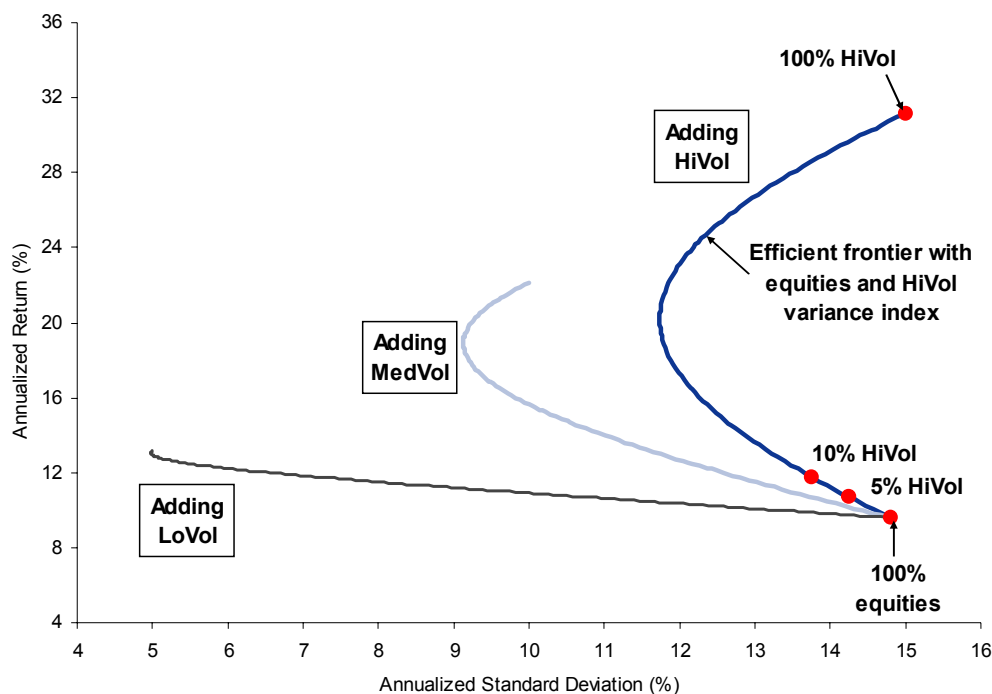
Exhibit 16 shows the risk, return, and Sharpe ratios of adding increasing weights of variance exposure on top of an S&P 500 portfolio. A substantial increase in risk-adjusted performance can be achieved with even modest allocations of variance:

- A portfolio 90% S&P 500 + 10% HiVol variance REP outperformed the S&P 500 215 bp per annum with a 104-bp reduction in risk. Sharpe ratios increased from 0.36 to 0.54.
- For investors with lower risk budgets, we found a portfolio 90% SPX + 10% LoVol REP outperformed an S&P 500 portfolio by 35 bp per annum with a 135-bp reduction in risk. Sharpe ratios increased from 0.36 to 0.42.

Exhibit 16: Adding 10% HiVol variance index exposure to an S&P 500 equity portfolio increased performance by 214 bp per annum, reduced volatility by 105 bp, and increased Sharpe ratios from 0.36 to 0.54

compound annual returns; annualized volatility of monthly returns January 31, 1996–September 30, 2007

Portfolio Weights		w/ HiVol Variance Index			w/ MedVol Variance Index			w/ LoVol Variance Index		
Equity	Variance Index	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio
100%	0%	9.60	14.80	0.36	9.60	14.80	0.36	9.60	14.80	0.36
99%	1%	9.81	14.69	0.37	9.72	14.68	0.37	9.63	14.66	0.36
98%	2%	10.03	14.58	0.39	9.85	14.55	0.38	9.67	14.53	0.37
97%	3%	10.24	14.47	0.41	9.97	14.43	0.39	9.70	14.39	0.37
96%	4%	10.46	14.36	0.43	10.10	14.31	0.40	9.74	14.26	0.38
95%	5%	10.67	14.26	0.44	10.22	14.19	0.41	9.77	14.12	0.39
⋮										
90%	10%	11.75	13.76	0.54	10.85	13.60	0.48	9.95	13.45	0.42



Source: Goldman Sachs Research estimates.

High risk premium leads to outperformance in hostile markets

We believe it is particularly important to understand the risks of volatility selling strategies in hostile markets. We therefore analyze their performance: (1) across the top 10 calendar-month declines in the S&P 500, (2) during the worst bear market in recent history (September 2000-September 2002), (3) conditional upon S&P 500 monthly performance, and (4) at their worst.

We find that, on average, volatility selling strategies tend to outperform in market declines as the upfront premiums serve as a downside cushion. That said, volatility strategies can fall sharply when risk re-prices dramatically. We analyze the drawdowns and emphasize de-levering variance positions using our REP technology to target manageable risk levels.

Returns across the top 10 calendar-month declines in the S&P 500

We show the 10 largest S&P 500 calendar-month declines from January 1996 to September 2007 and the corresponding returns for each of our volatility portfolios in Exhibit 17. The S&P 500 was down an average of 8.4% across the top 10 calendar-month declines, accompanied by an average 6.6-point increase in the VIX.

- **Variance selling strategies had the strongest average returns:** Although our HiVol variance index was calibrated for SPX-like volatility, it did not have SPX-like drawdowns in most cases, with an average monthly return of -1.9%. Option-based strategies outperformed the S&P 500 but suffered larger losses relative to variance. BXM was -6.5% on average, ATM puts -6.0%, and straddles -3.4%.
- The HiVol variance index had a positive return in 6 of the 10 months, while the BXM and PUT indices were down at least -1.3% in each month. The variance strategy did have the largest drawdown, however. During the Russian debt default (July-August 1998), the HiVol REP fell 15.3% vs. -14.5% for the S&P 500.
- Long S&P 500 and 60/40 equity-bond portfolios that replaced S&P 500 exposure with a 5% investment in the HiVol variance REP performed well. The 95% SPX/5% HiVol REP outperformed the long-only portfolio in 8 out of 10 months by an average of 30 bp.
- Note that call overwriting can underperform the S&P 500 in a down calendar month. In September 2001, the S&P 500 fell after 9/11 and the BXM rolled its strike at market lows. The S&P 500 then rallied 8%, causing the BXM to underperform.

Exhibit 17: Volatility strategy returns across the top 10 S&P 500 calendar month declines
January 1996–September 2007

Start Date	End Date	S&P 500 Return	VIX change	Variance Based Strategies			Option Based Strategies				Asset Allocation			
				HiVol 1M	MedVol 1M	LoVol 1M	CBOE BXM	CBOE PUT	Straddles	5% OTM Strangles	100% SPX	95% SPX + 5% HiVol	60/40 portfolio	55/40 + 5% HiVol
31-Jul-98	31-Aug-98	-14.5%	19.5	-15.3%	-10.2%	-4.9%	-11.8%	-10.3%	-9.0%	-6.6%	-14.5%	-14.5%	-6.8%	-6.9%
31-Aug-02	30-Sep-02	-10.9%	7.0	2.2%	1.5%	0.8%	-7.3%	-6.1%	-3.0%	-1.6%	-10.9%	-10.2%	-4.9%	-4.2%
31-Jan-01	28-Feb-01	-9.1%	6.3	-2.4%	-1.5%	-0.5%	-5.6%	-4.0%	0.4%	-0.6%	-9.1%	-8.8%	-4.7%	-4.4%
31-Aug-01	30-Sep-01	-8.1%	7.0	0.4%	0.5%	0.4%	-10.5%	-13.1%	-11.6%	-10.1%	-8.1%	-7.6%	-4.5%	-4.1%
31-Oct-00	30-Nov-00	-7.9%	6.0	0.8%	0.8%	0.7%	-2.9%	-1.3%	4.0%	2.0%	-7.9%	-7.4%	-3.5%	-3.0%
30-Jun-02	31-Jul-02	-7.8%	6.6	-8.7%	-5.9%	-2.9%	-8.3%	-8.8%	-8.0%	-6.4%	-7.8%	-7.8%	-3.5%	-3.5%
31-May-02	30-Jun-02	-7.1%	5.4	0.6%	0.5%	0.3%	-5.4%	-6.3%	-3.2%	-1.5%	-7.1%	-6.7%	-3.5%	-3.1%
28-Feb-01	31-Mar-01	-6.3%	0.3	0.3%	0.4%	0.4%	-5.6%	-5.2%	-3.3%	-2.1%	-6.3%	-6.0%	-4.1%	-3.8%
31-Jul-01	31-Aug-01	-6.3%	3.3	5.0%	3.5%	1.9%	-3.8%	-1.8%	0.7%	0.5%	-6.3%	-5.7%	-2.9%	-2.4%
31-Mar-02	30-Apr-02	-6.1%	4.5	-1.6%	-1.0%	-0.4%	-4.0%	-3.1%	-0.8%	0.0%	-6.1%	-5.8%	-2.0%	-1.8%
Average		-8.4%	6.6	-1.9%	-1.2%	-0.4%	-6.5%	-6.0%	-3.4%	-2.6%	-8.4%	-8.1%	-4.0%	-3.7%
Min		-14.5%	0.3	-15.3%	-10.2%	-4.9%	-11.8%	-13.1%	-11.6%	-10.1%	-14.5%	-14.5%	-6.8%	-6.9%
Max		-6.1%	19.5	5.0%	3.5%	1.9%	-2.9%	-1.3%	4.0%	2.0%	-6.1%	-5.7%	-2.0%	-1.8%

Source: CBOE, Ibbotson, Goldman Sachs Research estimates.

Bear market performance (September 2000 to September 2002)

Nine out of the top 10 SPX calendar-month declines in our study occurred between August 31, 2000, and September 30, 2002. In fact, this period was the worst bear market since the Great Depression as the market experienced the bursting of Tech Bubble, the effects of the 9/11 attacks, and the corporate accounting scandals of 2002, causing the S&P 500 Index to decline 45% in two years.

While the S&P 500 was down 45% over the period, the HiVol variance REP was up 45% and achieved that performance with a volatility 4 points lower than the market, 14% versus 18%.

Between September 2000 and September 2002, S&P 500 month returns were only positive 36% of the time, with an average monthly return of -2.2%. In contrast, the HiVol REP was profitable 72% of the time, with an average monthly return of 1.6%.

The BXM and Put indices outperformed the market by more than 15% during the pullback but were still down 30% and 28%, respectively, with volatility on par with the HiVol index. Of the option-based strategies, only straddles and strangles had positive returns.

Exhibit 18: Bear market performance: S&P 500 variance and option based strategies outperformed from August 31, 2000, to September 30, 2002

Monthly Returns	S&P 500 Return	Variance Based Strategies			Option Based Strategies				Asset Allocation			
		HiVol 1M	MedVol 1M	LoVol 1M	CBOE BXM	CBOE PUT	Straddles	5% OTM Strangles	100% SPX	95% SPX + 5% HiVol	60/40 portfolio	55/40 + 5% HiVol
Average	-2.2%	1.6%	1.2%	0.7%	-1.3%	-1.2%	0.2%	0.1%	-2.2%	-2.0%	-1.0%	-0.8%
Min	-10.9%	-8.7%	-5.9%	-2.9%	-10.5%	-13.1%	-11.6%	-10.1%	-10.9%	-10.2%	-4.9%	-4.4%
Max	7.8%	9.6%	6.6%	3.4%	3.5%	3.4%	4.5%	3.0%	7.8%	7.7%	3.0%	3.3%
% positive	36%	72%	72%	72%	48%	52%	60%	72%	36%	36%	40%	44%
Cumulative return	-44.7%	44.6%	32.5%	20.2%	-30.2%	-27.6%	1.9%	1.5%	-44.7%	-41.9%	-22.0%	-18.1%
Annualized return	-24.8%	19.4%	14.5%	9.2%	-15.8%	-14.3%	0.9%	0.7%	-24.8%	-22.9%	-11.2%	-9.1%
Annualized volatility	18.0%	14.0%	9.5%	4.9%	14.3%	14.9%	14.0%	10.2%	18.0%	17.4%	9.0%	8.5%

Source: CBOE, Ibbotson, Goldman Sachs Research estimates.

Performance conditional on S&P 500 monthly returns

Across the 140 calendar months in our study, we found a one-standard deviation monthly SPX move to be approximately 4%. In Exhibit 19, we analyze the average performance for the volatility indices conditional on S&P 500 returns being in 4 buckets: (1) return < -4%; (2) -4% <= return <= 0; (3) 0 < return <= 4%; (4) return > 4%.

- **SPX down over 4% (1 standard deviation move):** Variance and options selling strategies outperformed the market, with the LoVol REP performing the best. Consistent with our prior research, long forward variance strategies had strong positive returns.
- **SPX down 0 to 4%:** The S&P 500 was down 1.9% on average and the variance indices had a positive average return of 30 bp. Straddle selling did best.
- **SPX up 0 to 4%:** All strategies were positive on average, but the HiVol variance index had double the return of the Put and BXM strategies with an average outperformance of 180 bp versus straddles, 140 bp versus the market.
- **SPX up over 4%:** All the volatility strategies significantly underperformed the market. The HiVol REP and the BXM had the best performance among vol strategies.