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A Four-Factor Model for the Size, Value, and Profitability Patterns in Stock Returns

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Abstract

A four-factor model directed at capturing the size, value, and profitability patterns in average stock returns is rejected on the GRS test, but for applied purposes it seems to provide an acceptable description of average returns. The profitability patterns in average returns are less of a challenge for the model than the value patterns. The success of the factors does not seem to be sensitive to the way they are defined, at least for the definitions considered here.

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There is lots of evidence that average stock returns are related to the book-to-market equity ratio, *B/M*. There is also evidence that profitability adds to the description of average returns provided by *B/M*, and there is (weaker) evidence that investment is an additional dimension of average returns. The logic for why these three variables are related to average returns can be explained via the dividend discount model, which is the simplest valuation model for the market value of a firm's common stock.

In the dividend discount model, the market value of a share of a firm's stock is the present value of expected future dividends per share,

$$(1) \qquad m_t = \sum_{\tau=1}^{\infty} \mathrm{E}(d_{t+\tau}) / (1+r)^{\tau} .$$

In this equation m_t is the share price at time t, $\mathrm{E}(d_{t+\tau})$ is the expected dividend per share in period $t+\tau$, and r is (approximately) the long-term average expected stock return or, more precisely, the internal rate of return on expected dividends. Miller and Modigliani (1961) show that that the total time t value of the firm's outstanding stock implied by equation (1) is,

$$(2) \qquad M_t = \sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau} .$$

In this equation Y_t, is total equity earnings and $dB_t = B_t - B_{t-1}$ is the change in total book equity. Dividing by time t book equity gives,

$$(3) \qquad \frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau}}{B_t} .$$

Equation (3) makes three statements about expected stock returns. First, fix everything in equation (3) except the current value of the stock, M_t, and the expected stock return, r. The equation then tells us that a lower value of M_t (and thus a lower market-to-book ratio, M_t/B_t) implies a higher expected stock return, r. Equivalently, a higher book-to-market equity ratio, B_t/M_t, implies a higher expected stock return, r. This is the rationale for using the book-to-market ratio as a proxy for expected return.

Next, suppose we fix M_t and the values of everything else in equation (3) except expected future earnings and the discount rate (the expected stock return). The equation then tells us that higher expected future earnings imply a higher expected stock return. This is the motivation for tests of a positive relation between expected stock return and expected profitability.

The final implication of equation (3) is that for fixed values of B_t, M_t, and expected earnings, higher expected growth in book equity due to reinvestment of earnings implies lower expected return. This is the rationale for a negative predicted relation between expected stock return and expected investment.

Why has it been difficult to document profitability and investment effects in average returns? The only directly observable variable in (3) is the book-to-market ratio, B_t/M_t. This is perhaps why it has been easy to document the relation between B_t/M_t and average return. In contrast, we do not know the sequence of expected future earnings and expected investments in (3), and empirical work requires proxies. This is the stumbling block in research on the links between expected returns and profitability or investment. A recent paper by Novy-Marx (2012) does a better job on proxies for expected profitability, and it documents a strong relation between his profitability proxy and average returns.

Given the Novy-Marx (2012) results, it is appropriate to examine whether the three-factor model of Fama and French (1993) should be augmented to include a profitability factor. This paper examines the performance of candidate factors.

I. Empirical Asset Pricing Models

The three-factor model of Fama and French (1993) is an empirical asset pricing model. Standard asset pricing models work forward, from assumptions about investor tastes and portfolio opportunities to predictions about how risk should be measured and the relation between risk and expected return. Empirical asset pricing models work backward. They take as given the patterns in average returns, and propose models to capture the observed patterns. (If a theoretical license for this approach is needed, one can lean on Merton 1973 and Ross 1976.)

For example, the three-factor model of Fama and French (1993) is an attempt to capture the relation between average return and *Size* (market capitalization, price times shares outstanding) and the relation between average return and price ratios like the book-to-market ratio (*B/M*), which were the two well-known patterns in average returns at the time of our 1993 paper. The model is,

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + e_{it}. \quad (4)$$

In this equation R_{it} is the return on security or portfolio i for period t, R_{Ft} is the riskfree return for t, R_{Mt} is the return on the value-weight (VW) market portfolio, SMB_t is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks, and HML_t is the difference between the returns on diversified portfolios of high and low *B/M* stocks. If the sensitivities b_i, s_i, and h_i to the three portfolio returns in (4) suffice to capture all variation in expected returns, the true value of the intercept a_i is zero for all securities and portfolios i.

Novy-Marx (2012) provides evidence that, as predicted by the dividend discount model (3), there is variation in average returns related to profitability that is missed by the three-factor model. The issue of interest is then: can we capture the unexplained variation in average returns by adding a profitability factor to the model?

II. The Playing Field

The first step is to examine the *Size*, *B/M*, and profitability patterns in average returns we seek to explain. We begin with portfolios of stocks sorted on *Size* and *B/M* or *Size* and profitability. We then turn to portfolios of stocks triple-sorted on *Size*, *B/M*, and profitability.

Table 1 shows average excess returns (returns in excess of the one-month U.S. Treasury bill rate) for 25 VW portfolios from independent sorts of stocks into five *Size* groups and five *B/M* groups. (We call them 5x5 *Size*-*B/M* sorts.) The sample is all NYSE, Amex, and NASDAQ stocks on both CRSP and Compustat with share codes of 10 or 11 and the data required to compute *B/M* and profitability, but the *Size* and *B/M* breakpoints use only NYSE stocks. The period is July 1963 to December 2012. Fama and

French (1993) use the 25 portfolios in Table 1 to evaluate the three-factor model, and the patterns in average returns in Table 1 are like those reported in the earlier paper, but with 21 years of new data.

Specifically, in each *B/M* column of Table 1, average return typically falls from small stocks to big stocks – the size effect. The first column is the only exception. The glaring problem in this extreme growth column is the low average return of the smallest stocks, "microcaps". For the remaining four portfolios in the low *B/M* column, there is no obvious relation between *Size* and average return.

The relation between average return and *B/M*, called the value effect, shows up more consistently in Table 1. In every *Size* row, average return increases with *B/M*. As is now well-known, the value effect is stronger among small stocks. For example, for the tiny stocks in the first row, the average excess return rises from 0.19% per month for the lowest *B/M* portfolio (extreme growth stocks) to 1.11% per month for the highest *B/M* portfolio (extreme value stocks), a spread of 0.91%. In contrast, the average spread among the biggest stocks ("megacaps") is only 0.16%.

Table 1 also summarizes the number and market cap of the stocks in each of the 25 *Size*-*B/M* portfolios. Our *Size* breaks allocate 20% of NYSE firms to each row. Amex and NASDAQ stocks are usually a large fraction of our sample, however, and they tend to be small relative to the NYSE breaks. As a result, most sample stocks are in the smaller rows of Table 1. On average, 56.0% of the sample is in the microcap row, 82.1% is in the first three rows, and only 8.6% of the sample is in the megacap row. The story reverses, however, if we focus on aggregate market cap. Microcaps are tiny relative to stocks in the larger *Size* groups, so although they are more than half the sample, on average they are only 2.9% of total market cap. At the other extreme, on average megacaps account for less than 10% of the stocks, but they are almost three quarters of total market cap.

Among microcaps, the extreme growth and extreme value portfolios contain more stocks and more market cap than the other three *B/M* portfolios. In the remaining four rows of the matrix, the number of stocks and the percent of market cap in a portfolio decline monotonically from extreme growth to extreme value. In other words, our NYSE breakpoints create a bias toward growth in all but the microcap group. For example, the megacap extreme growth portfolio contains 3.2% of sample stocks and

32.6% of total market cap, versus 0.7% of stocks and 4.8% of total market cap for the megacap extreme value portfolio. This is not surprising: extreme value stocks tend to be firms that have done poorly, which means they are not likely to qualify as megacaps.

Table 2 shows average excess returns and other summary statistics for 25 VW portfolios from independent sorts of stocks into five *Size* quintiles and five profitability quintiles. The details of these 5x5 sorts are the same as in Table 1, except that the second sort is on profitability rather than *B/M*. For portfolios formed in June of year *t,* profitability (measured with accounting data for the fiscal year ending in *t*-1) is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity. We refer to this variable as operating profitability, *OP*, but it is actually operating profitability minus interest expense. Like the *Size* and *B/M* breakpoints, the *OP* breakpoints use only NYSE firms.

The patterns in average returns for the 25 *Size*-*OP* portfolios in Table 2 are like those observed for the 25 *Size*-*B/M* portfolios in Table 1. Holding operating profitability roughly constant, average return typically falls as *Size* increases. The decline in average return with increasing *Size* is rather smooth in the three middle columns of Table 2, but for the extreme low and high *OP* quintiles, the action with respect to *Size* is focused on lower average returns for megacaps.

The profitability effect identified by Novy-Marx (2012) is also evident in Table 2. In every row of the average return matrix, that is, for every *Size* group, extreme high operating profitability is associated with higher average return than extreme low operating profitability.

Table 2 also summarizes the allocations of stocks to the 25 *Size*-*OP* portfolios. Most striking is the large fraction of extreme low profitability firms in the microcap row of the *Size*-*OP* matrix. There are more than three times as many stocks and twice as much market cap in the extreme low profitability microcap portfolio (upper left corner of the matrix) as in any of the other microcap portfolios, and on average almost 80% of the stocks in the lowest *OP* group are microcaps. In contrast, among large stocks high operating profitability is more common. For example, the least profitable megacaps are on average 1.0% of sample stocks and 5.4% of total market cap, but extremely profitable megacaps are on average

2.3% of sample stocks and an impressive 25.8% of total market cap. This is not surprising: megacap stocks tend to be firms that have done extremely well, which means they are likely to be highly profitable.

The *Size*-*B/M* and *Size*-*OP* portfolios in Tables 1 and 2 do not disentangle the value and profitability effects in average returns. Tble 3 presents summary statistics for portfolios sorted jointly on *Size*, *B/M*, and *OP*. These allow us to examine how average returns vary with profitability holding *B/M* roughly constant and how average returns vary with *B/M* holding profitability roughly constant.

There are two problems. First, adding another sort can greatly expand the number of portfolios. For example, 5x5x5 sorts on *Size*, *B/M*, and *OP* produce 125 poorly diversified portfolios. To limit portfolio proliferation, we examine only two *Size* groups (small and big, using the median market cap for NYSE stocks as the breakpoint), four *B/M* quartiles, and four *OP* quartiles, a total of 2x4x4 = 32 portfolios. The second problem is that *B/M* and *OP* are negatively correlated, especially among big stocks; value stocks tend to have low profitability, which means portfolios of stocks with high *B/M* and high *OP* are likely to be poorly diversified. In fact, when we sort stocks independently on *Size*, *B/M*, and *OP*, the portfolio of big stocks in the highest quartiles of *B/M* and *OP* is often empty before July 1974. The problem is reduced when the sorts on *B/M* and *OP* use separate NYSE breakpoints for small and big stocks, and this is the approach taken here.

Table 3 shows average returns for the resulting 32 *Size*-*B/M*-*OP* portfolios. For small and big stocks, there is a clear value effect in every profitability quartile: holding operating profitability roughly constant, average return increases monotonically with *B/M*. Likewise, for both *Size* groups, there is a clear profitability effect in every *B/M* quartile: holding *B/M* roughly constant, average return increases monotonically with *OP*. Of note is the negative average excess return, -0.04% per month, on the small stock portfolio of extreme low *B/M* and extreme low *OP* stocks (small, low profitability, extreme growth stocks). This return is far lower than those of the other small stock portfolios and it is a strong challenge for the empirical asset pricing models considered later.

Table 3 also shows the average number of stocks and average percent of total market cap in each of the 32 *Size*-*B/M*-*OP* portfolios. Among big stocks *B/M* and *OP* are negatively correlated: growth stocks with low *B/M* tend to have high profitability and value stocks with high *B/M* tend to have low profitability. Most striking, on average 8.3 big stocks are in both the highest *B/M* quartile and the highest profitability quartile, versus 124.6 in the lowest *B/M* quartile and the highest *OP* quartile. Big stocks in the highest *B/M* and profitability quartiles account for a puny 0.9% of total market cap, versus 21.6% for big stocks in the lowest *B/M* and highest *OP* quartiles. In short, the negative correlation between *B/M* and profitability clearly limits the extent to which one can form diversified portfolios of big stocks with tilts toward value and high profitability.

The negative correlation between *B/M* and *OP* also shows up among small stocks in the three highest quartiles of profitability. In the lowest profitability quartile, however, there are on average more stocks and more total market cap in the lowest *B/M* quartile than in the highest *B/M* quartile. Thus, confirming the earlier evidence of Fama and French (1995), among small stocks the combination of extreme low profitability and extreme low *B/M* is common.

III. Factor Definitions

Tables 1 to 3 document patterns in average returns related to *Size*, *B/M*, and operating profitability. The next step is to develop an empirical asset pricing model that attempts to capture these patterns. We consider modifications to the three-factor model of Fama and French (1993), including changes in the definitions of the *Size* and value factors and the addition of a profitability factor.

We use a consistent naming convention to identify the candidate factors. There are four factors in each set: $R_M - R_F$, the excess return on the market; *SMB*, for small minus big; *HML*, for high minus low *B/M*; and *RMW*, for robust minus weak profitability. To construct *SMB*, *HML*, and *RMW* – which are described in more detail below – we assign stocks (independently) to two *Size* groups, two or three *B/M* groups, and two or three profitability groups. The intersections of the groups are value-weight portfolios, which we label with two or three letters. The first letter always describes the *Size* group, either small (*S*) or big (*B*). If we sort on *B/M*, the second character is the *B/M* group, low (*L*), neutral (*N*), or high (*H*). If we sort on *OP* but not *B/M*, the second character is the profitability group, robust (*R*), neutral (*N*), or weak

(W). When we sort on all three variables, the third character is the *OP* group. For example, in the sorts on *Size* and *B/M* but not *OP*, *SH* is small stocks with high *B/M*; in the *Size*-*OP* (but not *B/M*) sorts, *BW* is a portfolio of big stocks with weak profitability; and in the three-pass sorts, *BLR* is big stocks with low *B/M* and robust profitability.

To identify the factors produced by each set of portfolios, we use subscripts that describe the number of groups in each sort. The sequence is again *Size*, *B/M*, and *OP*. Thus, SMB_{233} is the *Size* factor constructed with two *Size*, three *B/M*, and three *OP* groups, HML_{22} is the value factor produced by two *Size* and two *B/M* groups, and RMW_{23} is the profitability factor from two *Size* and three *OP* groups.

Table 4 summarizes the factor definitions. We start by adding a profitability factor to the original three factors of Fama and French (1993). The *Size* and value factors of that model are constructed by sorting stocks into two *Size* groups and three *B/M* groups. The *Size* breakpoint is the NYSE median market cap, and the *B/M* breakpoints are the 30^{th} and 70^{th} percentiles of *B/M* for NYSE stocks. The unconditional sorts produce six VW portfolios. The *Size* factor $SMB_{23,B/M}$ is the average of the three small stock portfolio returns minus the average of the three big portfolio returns,

$$SMB_{23,B/M} = (SH + SN + SL) / 3 - (BH + BN + BL) / 3. \tag{5}$$

The value factor HML_{23} is the average of the two high *B/M* portfolio returns minus the average of the two low *B/M* portfolio returns. Equivalently, it is the average of value factors constructed with portfolios of only small and only big stocks,

$$\begin{aligned} HML_{23} &= (SH + BH) / 2 - (SL + BL) / 2 \\ &= [(SH - SL) + (BH - BL)] / 2 = (HML_{23,S} + HML_{23,B}) / 2. \end{aligned} \tag{6}$$

The profitability factor we add to the original three, RMW_{23}, is constructed in the same way as HML_{23} except the second sort is on operating profitability rather than the ratio of book equity to market equity. Specifically, RMW_{23} is the average of the two robust profitability returns minus the average of the two weak profitability returns, and it is also the average of profitability factors for small and big stocks,

$$\begin{aligned} RMW_{23} &= (SR + BR) / 2 - (SW + BW) / 2 \\ &= [(SR - SW) + (BR - BW)] / 2 = (RMW_{23,S} + RMW_{23,B}) / 2. \end{aligned} \tag{7}$$

The 2x3 sorts on *Size* and operating profitability produce a second *Size* factor,

(8) $$SMB_{23,OP} = (SR + SN + SW) / 3 – (BR + BN + BW) / 3.$$

The *Size* factor from the 2x3 sorts, SMB_{23}, is the average of $SMB_{23,B/M}$ and $SMB_{23,OP}$,

(9) $$SMB_{23} = (SMB_{23,B/M} + SMB_{23,OP}) / 2.$$

Since HML_{23} and RMW_{23} weight small and big stock portfolio returns equally, both are roughly neutral with respect to size. Note, however, that HML_{23} is not neutral with respect to operating profitability and RMW_{23} is not neutral with respect to value versus growth. A similar argument implies $SMB_{23,B/M}$ is roughly neutral with respect to *B/M*, but not *OP*, and $SMB_{23,OP}$ is roughly neutral with respect to *OP*, but not *B/M*. Since the *Size* factor, SMB_{23}, is the average of $SMB_{23,B/M}$ and $SMB_{23,OP}$, it is not neutral with respect to either operating profitability or value versus growth.

When we developed the three-factor model, (5) and (6) were the only versions of *SMB* and *HML* considered. The choice of a 2x3 sort on *Size* and *B/M* is, however, arbitrary. To test the sensitivity of the asset pricing results to this choice, we construct versions of *SMB*, *HML*, and *RMW* from 2x2 rather than 2x3 sorts on *Size* and *B/M* or *Size* and *OP*, using the NYSE medians for *B/M* and *OP* as the breakpoints. The definitions of HML_{22} and RMW_{22} are analogous to the definitions of HML_{23} and RMW_{23} in (6) and (7), and the *Size* factor for this set is,

(10) $$\begin{aligned} SMB_{22} &= [(SH + SL – BH – BL) / 2 + (SR + SW – BR – BW) / 2] / 2 \\ &= (SMB_{22,B/M} + SMB_{22,OP}) / 2. \end{aligned}$$

The next candidate factors use three sorts to jointly control for *Size*, *B/M*, and *OP*. The first version uses 2x2x2 sorts. Specifically, we sort stocks independently into two *Size* groups, two *B/M* groups, and two *OP* groups using NYSE medians as breakpoints. The intersections of the groups are eight VW portfolios. Table 4 describes the factors constructed with the portfolios. The *Size* factor SMB_{222} is the average of the returns on the four small stock portfolios minus the average of the returns on the four big stock portfolios. The value factor HML_{222} is the average return on the four high *B/M* portfolios minus the average return on the four low *B/M* portfolios. And the profitability factor RMW_{222} is the average return on the four robust profitability portfolios minus the average return on the four weak profitability portfolios. We can again interpret the value factor HML_{222} as the average of the returns on small and big stock value factors,

(11) $$HML_{222} = (HML_{222,S} + HML_{222,B}) / 2.$$

Similarly, RMW_{222} is the average of the returns on small and big stock profitability factors,

(12) $$RMW_{222} = (RMW_{222,S} + RMW_{222,B}) / 2.$$

The next factors also use three independent sorts to control for *Size*, *B/M*, and *OP*, but the sorts are 2x3x3, with two *Size* groups, three *B/M* groups, and three *OP* groups. The *B/M* and *OP* breakpoints are the 30^{th} and 70^{th} percentiles of the variables for NYSE stocks. The three sorts produce 18 VW portfolios. As Table 4 describes, SMB_{233} is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios, HML_{233} is the difference between the average returns on the six high and six low *B/M* portfolios, and RMW_{233} is the difference between the average returns on the six robust and six weak profitability portfolios.

Since SMB_{222} and SMB_{233} weight high and low *B/M* portfolio returns equally and robust and weak *OP* portfolio returns equally, these *Size* factors are roughly neutral with respect to value and profitability. Similarly, HML_{222} and HML_{233} are roughly neutral with respect to *Size* and *OP*, and RMW_{222} and RMW_{233} are roughly neutral with respect to *Size* and *B/M*. We shall see, however, that neutrality with respect to a characteristic does not imply low correlation between factor returns.

Finally, Novy-Marx (2012) proposes value and profitability factors that use the 2x3 sorts of equations (5) through (8), except he adjusts each firm's *B/M* and *OP* by its industry mean before sorting and each stock in a portfolio is balanced by an offsetting short position in the stock's industry – in other words, industry hedges. Novy-Marx does not include a *Size* factor in his asset pricing model, but a *Size* factor turns out to be important in our tests. Thus, we supplement his value and profitability factors, HML_{NM} and RMW_{NM}, with the SMB_{23} of equation (9). This produces a direct comparison of his factors, HML_{NM} and RMW_{NM}, and HML_{23} and RMW_{23} of equations (6) and (7).

IV. Summary Statistics for Factor Returns

The general form of the four-factor model is

(13) $$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + e_{it},$$

which is the three-factor model (4) augmented by the profitability factor, *RMW*. Like the three-factor model, (13) is an empirical asset pricing model proposed as a parsimonious way to capture the patterns in average returns related to *Size*, *B/M*, and profitability. Before taking up this asset pricing challenge, however, we examine summary statistics for the different versions of the factors, in Table 5. Summary statistics for the returns on the portfolios used to construct the factors are in Appendix Table A1.

Average *SMB* returns are in a narrow range, from 0.28% per month (t = 2.30) to 0.31% per month (t = 2.59). The standard deviations of the *SMB* returns are also similar, from 2.91% to 3.11% per month. This is not surprising since the *Size* breakpoint in all the sorts is the NYSE median market cap. All the average *SMB* returns are more than 2.2 standard errors from zero.

The summary statistics for *HML* and *RMW* depend more on how the factors are constructed. For example, the average HML_{23} and RMW_{23} returns, from the separate 2x3 *Size*-*B/M* and *Size*-*OP* sorts, are a bit larger than the average HML_{NM} and RMW_{NM} returns from Novy-Marx' 2x3 sorts, but his factors have much lower volatility and thus much larger *t*-statistics. Lower volatility is an advantage in asset pricing if it is due to better diversification, but it may not help if it is due to the industry hedges in Novy-Marx' factors. The asset pricing tests will settle this issue.

The standard deviations of *HML* and *RMW* are lower when only two *B/M* or *OP* groups are used, due to better diversification. In the 2x2 and 2x2x2 sorts, *HML* and *RMW* include all stocks, but in the 2x3 and 2x3x3 sorts, the stocks in the middle 40% of *B/M* and *OP* are dropped. On the other hand, since the sorts that use three *B/M* or *OP* groups focus more on the extremes of the two variables, they produce larger average *HML* and *RMW* returns. The average *HML* return is 0.38% per month in the standard 2x3 *Size*-*B/M* sorts, versus 0.29% in the 2x2 sorts. In the 2x3x3 *Size*-*B/M*-*OP* sorts, the average *HML* return is 0.77% per month versus 0.36% per month in the 2x2x2 sorts. Similar differences are observed in average *RMW* returns. The *t*-statistics (and thus the Sharpe ratios) for average *HML* and *RMW* returns are, however, similar for sorts of stocks into two or three *B/M* and *OP* groups.

Average *HML* and *RMW* returns are also larger in the sorts that jointly control for *Size*, *B/M*, and *OP*. For example, the average return for the standard value factor, HML_{23}, is 0.38% per month (t = 3.22), versus 0.77% (t = 3.94) for HML_{233}, which adds a control for profitability. Similarly, the average RMW_{23} return is 0.26% per month (t = 2.93), versus 0.62% (t = 4.47) for RMW_{233}. The standard deviations of *HML* and *RMW* are, however, also much larger in the 2x3x3 sorts. This is a result of the negative correlation between *B/M* and *OP*, which causes the portfolios with high *B/M* and robust profitability to have relatively few stocks in the 2x3x3 sorts.

The value and profitability factors are averages of small and big stock factors. Like Table 1, Table 5 confirms earlier evidence that the value premium is larger for small stocks (e.g., Fama and French 1993, 2012). For example, in the 2x3 sorts on *Size* and *B/M*, the average $HML_{23,S}$ return is 0.55% per month (t = 4.10), versus 0.21% (t = 1.67) for $HML_{23,B}$. For all methods of construction, the average difference between the HML_S and HML_B returns is at least 2.5 standard errors above zero. In the 2x2 and 2x3 sorts on *Size* and *B/M*, the average HML_B return is less than 1.7 standard errors from zero. The average HML_B return is larger when the sorts control for profitability and *B/M*. In the 2x2x2 and 2x3x3 sorts on *Size*, *B/M*, and *OP*, the average HML_B returns, 0.25% and 0.57% per month, are 2.40 and 2.50 standard errors above zero. Controlling for profitability thus produces stronger evidence of a value premium among big stocks.

The profitability premium is also larger for small stocks (Table 5), but the average differences between the RMW_S and RMW_B returns are only 0.92 to 1.68 standard errors from zero. Thus, the evidence that the expected premium is larger for small stocks is rather weak.

Part B of Table 5 shows the correlation matrix for each set of factors. The *Size* factor, *SMB*, is positively correlated with the market return and negatively correlated with *HML* and *RMW*, but the correlations are within 0.32 of zero. The value and profitability factors, *HML* and *RMW*, are negatively correlated with the market, and the correlations, again unimpressive, are a bit stronger for *HML*. The correlations between the value and profitability factors are more interesting. When *HML* and *RMW* are from separate sorts on *Size* and *B/M* or *Size* and *OP*, the correlation is close to zero -0.04 for 2x2 sorts and 0.08 for 2x3 sorts. But when the sorts jointly control for *Size*, *B/M*, and profitability, the correlations between *HML* and *RMW* are 0.59 (2x2x2) and 0.66 (2x3x3). These results deliver on the earlier warning that, even after jointly controlling for variables, factor returns can be strongly correlated.

V. Model Performance

We now test how well different models explain average excess returns on the 25 *Size*-*B/M* portfolios of Table 1, the 25 *Size*-*OP* portfolios of Table 2, and the 32 *Size*-*B/M*-*OP* portfolios of Table 3. We consider four models: (i) the CAPM, in which the excess market return, $R_M - R_F$, is the only right hand side (RHS) variable; (ii) the three-factor model of Fama and French (1993), which adds *SMB* and *HML*; (iii), a three-factor model that substitutes the profitability factor *RMW* for *HML*; and (iv) the four-factor model (16), which uses all four variables. Five sets of factors are used, formed from: (i) separate 2x3 *Size*-*B/M* and *Size*-*OP* sorts; (ii) 2x2 sorts; (iii) 2x2x2 sorts that jointly control for *Size*, *B/M*, and *OP*; (iv) 2x3x3 sorts, and (v) the Novy-Marx factors.

With three sets of LHS portfolios and five sets of factors, there is reason to limit the results shown to models that fare relatively well. The first casualty is the CAPM. The CAPM cannot explain average returns on the 25 *Size*-*B/M* portfolios (Fama and French 1993, 2012), and it also fares poorly in tests on the 25 *Size*-*OP* and the 32 *Size*-*B/M*-*OP* portfolios, with, for example, average absolute intercepts more than twice those of other models. Skipping the details, the CAPM fails because the market betas of the LHS portfolios examined here are close to 1.0 and so cannot capture the value and profitability patterns in average returns. The Fama-French three-factor model does poorly when the LHS portfolios involve a sort on profitability. Thus, we show results for this model only when the LHS assets are the 25 *Size*-*B/M* portfolios. The three-factor model that substitutes *RMW* for *HML* fares poorly when the LHS portfolios use a sort on *B/M*, so we show results for this model only for the 25 *Size*-*OP* portfolios.

If a model completely captures expected returns, the regression slopes for the model's factors and the average returns on the factors combine to explain the average excess returns on all assets. In other words, the ideal model's regression intercepts are indistinguishable from zero for all left hand side (LHS) assets. Table 6 shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989), which tests this criterion, for combinations of LHS portfolios and factors, and the *p*-value of the *GRS* statistic, which is the probability of getting a *GRS* statistic smaller than the one observed if the true intercepts are all zero. A *p*-value near 1.0 says the model is almost surely an incomplete story for average returns. The table also shows the average absolute value of the 25 or 32 intercepts produced by the model, the average of the regression R^2, and the average standard error of the intercepts.

The *GRS* test rejects all models considered, for all LHS portfolios and RHS factors. In the tests on the 25 *Size*-*B/M* portfolios and the 32 *Size*-*B/M*-*OP* portfolios, all models are rejected with probability close to 1.0. The models fare a bit better on the 25 *Size*-*OP* portfolios, but only the four-factor model that uses the 2x2x2 versions of the *Size*, *B/M*, and *OP* factors produces a rejection (*p*-value = 0.942) below the 0.97 level. In general, the models we consider have lower average absolute intercepts and *GRS* statistics in the tests on the 25 *Size*-*OP* portfolios than in the tests on either the 25 *Size*-*B/M* portfolios or the 32 *Size*-*B/M*-*OP* portfolios. Portfolios formed on *B/M* are apparently a bigger challenge for asset pricing models than portfolios formed on operating profitability.

The *GRS* test compares the Sharpe ratios for the portfolio of RHS portfolios that has the highest Sharpe ratio and the portfolio of LHS and RHS portfolios that has the highest Sharpe ratio. The hypothesis that the RHS portfolios alone capture all variation in expected returns is rejected if adding the LHS assets produces a statistically reliable increase in the maximum Sharpe ratio. In solving for the maximum Sharpe ratios, no constraints on shortselling are imposed, and the weights on individual LHS and RHS portfolios are often wildly positive and negative (see Fama and French 2013). This is appropriate for tests of asset pricing models because we want to ferret out model problems in an unconstrained way. But for investors, rejection on the *GRS* test may be irrelevant if due to small deviations of average returns from model predictions. Our favorite statistics for evaluating a model for investment purposes are the average absolute intercept and, for more detail, the full matrix of intercepts, which shows how pricing errors are related to the characteristics of the LHS portfolios.

Asset pricing models are called models because they are simplified propositions that will be rejected in tests with power. As a result, we are less interested in whether competing models are rejected than in their relative performance, which we judge using *GRS* statistics, average absolute intercepts, and other metrics. We want to identify the model that is the best (but imperfect) story for average returns on portfolios formed in different ways.

Fama and French (1993) find that the *GRS* test rejects their three-factor model when it is confronted with the 25 *Size*-*B/M* portfolio returns it was designed to explain. Table 6 says that two decades of out of sample evidence do not change this conclusion. Although it is rejected, the model's average absolute intercept for the *Size*-*B/M* portfolios, 0.101, suggests it does a reasonable job capturing the patterns in average returns related to *Size* and *B/M*. The performance of the three-factor model is similar for all methods of factor construction. For example, the average absolute intercept rounds to 0.10 for all five definitions of *SMB* and *HML*. In the tests on the 25 *Size*-*B/M* portfolios, adding the profitability factor, *RMW*, to the three-factor model produces lower *GRS* statistics and slightly lower average absolute intercepts for all definitions of the factors. Later we examine the intercepts in detail to judge where the three-factor model fails to capture average returns on the 25 *Size*-*B/M* portfolios and to assess the improvements delivered by the four-factor model.

The three-factor model that substitutes the profitability factor *RMW* for *HML* has more success explaining average returns on the 25 portfolios formed from sorts on *Size* and operating profitability (Table 6B), and all factor definitions again produce similar results. Adding *HML* to the model has little effect on the *GRS* statistic and does not consistently improve the average absolute intercept. The *GRS* test again rejects every model as a complete description of the patterns in average returns on the 25 *Size*-*OP* portfolios, but the rejections are weaker and the average absolute intercepts are smaller than those produced by the 25 *Size*-*B/M* portfolios. For example, in the tests of the four-factor model on the 25 *Size*-*B/M* portfolios, the *GRS* statistics for the five different definitions of the factors range from 2.42 to 3.13

and the average absolute intercepts are between 0.089 and 0.097. In contrast, in the tests of the four-factor model on the 25 *Size-OP* portfolios, the *GRS* statistics are lower, 1.50 to 1.72, and the average absolute intercepts are from 0.059 to 0.096. It again appears that the patterns in average returns related to *Size* and *B/M* are a bigger challenge to asset pricing models than the patterns related to *Size* and operating profitability.

The 32 portfolios formed using 2x4x4 sorts expose the *Size*, *B/M*, and profitability patterns in average returns, and we need the four-factor model to capture them. As in all the tests, different versions of the factors deliver similar results in tests of the four-factor model. For example, the average absolute intercepts, in Table 6C, are in a narrow band from 0.102 to 0.118.

The *GRS* statistics for the four-factor model are smaller in the tests on the 32 *Size-B/M-OP* portfolios than in the tests on the 25 *Size-B/M* portfolios, but the average absolute intercepts are larger for the *Size-B/M-OP* portfolios. This apparent contradiction is caused by differences in the power of the tests. As noted in Table 3, the negative correlation between *B/M* and profitability means that sorting jointly on *B/M* and *OP* produces some poorly diversified portfolios. This poor diversification is at least partly responsible for lower average R^2 (0.82 to 0.85 versus 0.91 or 0.92) and higher average standard errors of the intercepts (0.09 to 0.11 versus 0.07) in the tests of the four-factor model on the 32 *Size-B/M-OP* portfolios. As a result, the power of the *GRS* test is lower and regression intercepts that are further from zero are more consistent with chance.

Finally, Novy-Marx' (2012) industry-controlled *HML* and *RMW* have much lower standard deviations than the other versions of *HML* and *RMW* (Table 5). Table 6 shows, however, that judged on the average absolute intercept, the four-factor model that uses his factors performs slightly better in the tests on the 25 *Size-B/M* portfolios, a bit worse in the tests on the 32 *Size-B/M-OP* portfolios, and much worse in the tests on the 25 *Size-OP* portfolios. His factors tend to do better in the *GRS* tests, but this is due to lower precision. Average R^2 is lower and the average standard error of the intercepts is higher with his factors. We infer that the industry hedges in Novy-Marx' factors lower the volatility of *HML* and *RMW* but at the expense of some loss of precision in asset pricing tests, at least for the LHS portfolios considered here. In his tests, Novy-Marx (2012) includes a momentum factor formed in the same way as his value and profitability factors. In our tests, adding his momentum factor to the four-factor model has modest effects, good and bad, on model performance. A momentum factor is surely important, however, when the LHS portfolios involve momentum sorts (for example, Fama and French 2012).

VI. Regression Details

For more perspective on model performance we examine details of the regression results. To keep the presentation manageable, we do not show results for the factors from the separate 2x2 *Size*-*B/M* and *Size*-*OP* sorts since they are close to those from the 2x3 sorts. We also exclude the 2x3x3 factors that jointly control for *Size*, *B/M*, and operating profitability since they produce results close to those from the 2x2x2 sorts. We always show results for the factors from the 2x3 *Size*-*B/M* and *Size*-*OP* sorts (we call them the standard 2x3 sorts) since they use the original approach to factor formation of Fama and French (1993). We also always show results for the factors from the 2x2x2 *Size*-*B/M*-*OP* sorts since they tend to perform a bit better in the summary results of Table 6, and we show results for the Novy-Marx factors in the tests on the 25 *Size*-*B/M* portfolios since they produce average absolute intercepts similar to those from the 2x2x2 factors. These choices are innocuous since different versions of the factors perform similarly in all the tests in Table 6.

A. *Size*-*B/M* Portfolios

In the Table 6 tests on the 25 *Size*-*B/M* portfolios, adding a profitability factor to the three-factor model improves the *GRS* statistic and the average absolute intercept. The gains produced by switching from the three- to the four-factor model are quite modest. For example, the largest improvement in the average absolute intercept is only 0.013% per month (1.3 basis points, 2x3x3 factors), and the smallest is 0.004% per month (2x3 and 2x2 factors). Examining the sources of the gains nevertheless provides insights into some of the well-known problems of the three-factor model.

Table 7 reports intercepts and pertinent slopes from the three- and four-factor regressions. The three-factor intercepts for the 25 *Size*-*B/M* portfolios in Part A of Table 7 show familiar patterns (Fama and French 1993, 2012). Extreme growth stocks (left column of the intercept matrix) are the big problem. The portfolios of small extreme growth stocks produce negative three-factor intercepts and the portfolios of large extreme growth stocks produce positive intercepts. Microcap extreme growth stocks (upper left corner of the intercept matrix) are a huge problem. The three-factor intercepts for this portfolio are -0.52% per month (t = -5.33, 2x3 factors), -0.40 (t = -3.92, Novy-Marx factors), and -0.46 (t = -4.79, 2x2x2 factors).

Switching to the four-factor model reduces the problems. When we use the standard factors, from the 2x3 sorts, the intercept for the microcap extreme growth portfolio rises to -0.34 (t = -3.86) in the four-factor model, and the intercepts for four of the five extreme growth portfolios shrink toward zero. Despite these improvements, the pattern in the extreme growth intercepts – negative for small stocks, positive for large – survives. Much the same behavior is observed in the four-factor intercepts for the extreme growth portfolios when the 2x2x2 factors are used.

The four-factor model that uses the Novy-Marx factors does better on extreme growth stocks. Most impressive, the switch from the three-factor to the four-factor model causes the intercept for the microcap extreme growth portfolio to fall from -0.40 (t = -3.92) to -0.16 (t = -1.58). But all the news is not good. The four-factor Novy-Marx intercepts for three of the other microcap portfolios, for example, go strongly positive and are more than 2.25 standard errors from zero. The four-factor models that use the standard 2x3 or 2x2x2 factors do not improve the intercepts for the extreme growth portfolios as much as the Novy-Marx factors, but the standard 2x3 factors and especially the 2x2x2 factors tend to do better in the other four columns of the four-factor intercept matrix. As a result, the four-factor average absolute intercept obtained with the Novy-Marx factors is just 0.008% (less than one basis point) lower than that produced by the standard 2x3 factors and 0.002% lower than that produced by the 2x2x2 factors.

Part B of Table 7 shows the four-factor slopes for *HML* and *RMW* when we use the three versions of the factors to explain the returns on the 25 *Size-B/M* portfolios. The market and *SMB* slopes are not shown. The market slopes are always close to 1.0 for all portfolios, and the *SMB* slopes are always strongly positive for small stocks and slightly negative for big stocks. The market and *SMB* slopes are not sensitive to factor definitions or the switch from the three-factor to the four-factor model, so they cannot account for the changes in the intercepts. Thus, to save space, here and later, we concentrate on the *HML* and *RMW* slopes that are more important for interpreting changes in the intercepts.

The *HML* slopes in Table 7 for the four-factor models that use the standard 2x3 factors or the 2x3 Novy-Marx factors are similar to those for the three-factor models (not shown), a result implied by the low correlation between *HML* and *RMW* (0.08 in Table 2) for the two versions of the factors. The changes in the intercepts observed in going from the three-factor to the four-factor model thus center on the *RMW* slopes. Table 7 shows that the microcap extreme growth portfolio has the most negative *RMW* slopes, -0.52 (t = -12.46) for the standard 2x3 version of *RMW* and -0.69 (t = -7.85) for the Novy-Marx version. Thus, despite being classified as extreme growth stocks on *B/M*, these firms have a strong tilt toward low profitability, which accounts for the improvements in the intercepts from the four-factor regressions. In contrast, the megacap extreme growth portfolio (bottom left corner of the matrix) has the most strongly positive *RMW* slopes, 0.22 (t = 9.99) for the 2x3 version of *RMW* and 0.38 (t = 7.43) for the Novy-Marx version. This strong positive profitability tilt then explains why adding *RMW* to the model shrinks the megacap extreme growth portfolio's intercept.

The average *HML* and *RMW* returns for the standard 2x3 factors and the 2x2x2 factors are close to those for the Novy-Marx factors (Table 5), but the Novy-Marx factors produce more extreme negative *HML* and *RMW* slopes for the microcap extreme growth portfolio. As a result, this portfolio's four-factor intercept is closer to zero when we use the Novy-Marx factors. We know from Table 6, however, that judged on the average absolute intercept, the overall performance of the Novy-Marx factors in the tests of the four-factor model on the 25 *Size-B/M* portfolios is not much better than that of the standard 2x3 factors, it is near identical to that of the 2x2x2 factors, and as noted above, there are portfolios for which the 2x3 factors and the 2x2x2 factors better describe average returns.

The four-factor *HML* slopes in Table 7B show a familiar pattern, strongly negative for extreme growth portfolios increasing to strongly positive for extreme value portfolios. The patterns in the *RMW* slopes are less consistent, especially for the standard 2x3 and Novy-Marx factors. The *RMW* slope is negative for all microcap portfolios and it is more negative for microcap growth portfolios. This is in line with the evidence in Fama and French (1995) that the small group is afflicted with a large dose of extremely unprofitable growth stocks. The *RMW* slope is strongly positive for the megacap extreme growth portfolio and strongly negative for the megacap extreme value portfolio, which is in line with our earlier evidence that among large firms extreme growth stocks are more profitable than extreme value stocks. When the standard 2x3 or Novy-Marx factors are used, this intuitive pattern in the *RMW* slopes shows up only among megacaps. This may seem surprising, but there is an explanation. With separate 2x3 (standard or Novy-Marx) sorts, there is smearing of profitability and value effects in *HML* and *RMW* because the sorts on *Size* and *B/M* that produce *HML* do not control for profitability and the sorts on *Size* and *OP* that produce *RMW* do not control for *B/M*.

The factors from the 2x2x2 sorts jointly control for *Size*, *B/M*, and *OP* and so should better disentangle value and profitability exposures. Part B of Table 7 shows that in the tests on the 25 *Size-B/M* portfolios, the patterns in the *RMW* slopes are indeed clearer when the 2x2x2 factors are used. The biggest difference is that with the 2x2x2 factors, all the *RMW* slopes in the two rightmost columns of the 5x5 matrix are strongly negative, which is in line with the fact that (microcaps aside) value stocks tend to be less profitable. With all three sets of factors, *RMW* exposure for megacaps goes from strongly positive for low *B/M* portfolios to strongly negative for high *B/M* portfolios, but the 2x2x2 factors also produce a weaker version of this intuitive relation in the middle three *Size* groups.

B. *Size-OP* Portfolios

The *GRS* statistics in Table 6 say that the four-factor model and the three-factor model that drops *HML* provide comparable descriptions of average returns on the 25 portfolios formed on *Size* and operating profitability. The average absolute four-factor intercept for the Novy-Marx factors, 0.096, is much larger than the average, 0.063, for both the standard 2x3 factors and the 2x2x2 factors. Thus, Part A of Table 8 shows the four-factor intercepts for the 2x3 and 2x2x2 factors, but not for the Novy-Marx factors. Part B shows the *HML* and *RMW* slopes obtained with the 2x2x2 factors, which again provide a clean picture of the value and profitability tilts of the portfolios.

The four-factor intercepts for the 25 *Size-OP* portfolios in Table 8A show no particular patterns and are mostly close to zero, which is not surprising given that the average absolute intercepts are much smaller for the *Size-OP* portfolios than for the *Size-B/M* portfolios. The highest profitability microcap and megacap portfolios (upper right and lower right corners of the matrices) produce the most extreme intercepts, for example, -0.18 (t = -2.41) and 0.12 (t = 2.47) in the tests that use the factors from the 2x2x2 sorts on *Size*, *B/M*, and *OP*. In general, the four-factor model seems to be a good description of average returns on the 25 *Size-OP* portfolios. Again, portfolios formed with 5x5 sorts on *Size* and profitability seem to pose less of a challenge to asset pricing models than 5x5 sorts on *Size* and *B/M*.

The *HML* slopes for the 25 *Size-OP* portfolios, in Table 8B, show a clear pattern among megacaps – strongly positive for the lowest profitability megacaps and strongly negative for megacaps with the highest profitability. This confirms that among megacaps, low profitability is associated with value and high profitability is associated with growth. The negative correlation between *HML* slopes and profitability gets progressively weaker as *Size* decreases. For microcaps, the lowest and highest profitability portfolios have *HML* slopes close to zero, which suggests little tilt toward value or growth.

As expected, the *RMW* slopes for the 25 *Size-OP* portfolios increase monotonically from strongly negative for low profitability portfolios to strongly positive for high profitability portfolios. In the two smallest *Size* quintiles, negative exposure to *RMW* is limited to the lowest profitability quintile. In the next two *Size* quintiles, negative exposure to *RMW* extends to the second profitability quintile, and among megacaps, the three lowest profitability quintiles have strong negative *RMW* exposure.

C. *Size-B/M-OP* Portfolios

Part A of Table 9 shows the four-factor intercepts obtained when the LHS portfolios are the 32 *Size-B/M-OP* portfolios formed from the 2x4x4 sorts. The RHS returns are the standard factors from the separate 2x3 *Size-B/M* and *Size-OP* sorts and the factors from the joint 2x2x2 sorts on *Size*, *B/M*, and *OP*.

The four-factor model best captures average returns on the 32 *Size-B/M-OP* portfolios, and Table 6 says that the two versions of the factors used in Table 9 are largely equivalent in terms of *GRS* statistics and average absolute intercepts. Table 9 says that the two versions of the factors produce similar intercepts and identify the same model problems.

The biggest problem is the portfolio of small stocks with the lowest profitability and lowest *B/M* (small, low profitability, extreme growth stocks). For this portfolio, the intercepts produced by the two versions of the factors are -0.38% per month (t = -4.06) and -0.41% per month (t = 4.02). This confirms the inference from the tests on the 25 *Size-B/M* portfolios (Table 7) that small low-profitability growth stocks are the four-factor model's big problem. Table 9 also says that small stocks with the lowest *B/M* and the highest profitability may be a problem for the four-factor model. The intercepts for this portfolio for the two versions of the factors are -0.16 (t = -3.34) and -0.12 (t = -2.02).

The portfolios of small and big stocks with the highest *B/M* and the highest *OP* (highly profitable extreme value stocks) produce extreme four-factor intercepts (positive for small stocks and negative for big stocks), but they are only -2.08 to 1.47 standard errors from zero, suggestive of a chance result. The imprecision of the intercepts for these portfolios is due to poor diversification, noted earlier (Table 3): highly profitable extreme value stocks are rare, especially among big stocks. Part B of Table 8 confirms that the regression R^2 for these portfolios are low, 0.57 for big stocks and 0.67 for small stocks. The regression R^2 are also low for other portfolios that combine high *B/M* with high profitability, but these portfolios produce intercepts closer to zero. Finally, another manifestation of the power problem, noted earlier (Table 5), is that the 32 *Size-B/M-OP* portfolios produce weaker rejections on the GRS test but larger average absolute intercepts than the 25 *Size-B/M* portfolios, which are better diversified.

Part B of Table 9 shows the *HML* and *RMW* slopes for the 32 *Size-B/M-OP* portfolios produced by the factors from the 2x2x2 sorts on the three variables. For small and big stocks and for all *RMW* quartiles, the *HML* slopes increase from strongly negative for the low-*B/M* portfolios to strongly positive for high-*B/M* portfolios. Likewise, for both *Size* groups and all four *B/M* groups, the *RMW* slopes increase from strongly negative for the low profitability portfolios to strongly positive for the high *OP* portfolios. None of this is surprising, but it again shows that the factors from the 2x2x2 sorts, which jointly control for *Size*, *B/M*, and profitability cleanly separate value and profitability exposures in returns.

VII. Conclusions

There are patterns in average returns related to *Size*, *B/M*, and operating profitability. The *GRS* test easily rejects a parsimonious four-factor model directed at capturing these patterns, but for most applications the model seems to provide acceptable descriptions of average returns on 25 *Size*-*B/M* portfolios, 25 *Size*-*OP* portfolios, and 32 *Size*-*B/M*-*OP* portfolios formed to expose the three patterns in average returns. The four-factor intercepts for the 25 *Size*-*OP* portfolios show no patterns and almost all are close to zero. The four-factor model is not quite as successful with the average returns on the 25 *Size*-*B/M* portfolios. The model alleviates some of the well-known problems of the three-factor model of Fama and French (1993) and delivers intercepts for the *Size*-*B/M* portfolios that are generally close to zero, but the *GRS* statistics are always higher for the *Size*-*B/M* portfolios than for the *Size*-*OP* portfolios, and the average absolute intercepts are almost always higher for the *Size*-*B/M* portfolios. These results suggest that the value patterns in average returns are more challenging for empirical asset pricing models than the profitability patterns.

It is interesting that all five sets of factors we consider – (i) separate 2x3 sorts on *Size* and *B/M* or *Size* and *OP*, (ii) separate 2x2 sorts, (iii) 2x2x2 sorts that jointly control for *Size*, *B/M*, and *OP*, (iv) 2x3x3 sorts, and (v) the Novy-Marx factors – provide similar descriptions of average returns on the LHS portfolios examined. In the jargon of asset pricing, as long as the factor portfolios are well-diversified and are formed to produce spreads on the characteristics that seem to be related to expected returns, the spanning properties of factors do not seem to be sensitive to details of the way the factors are defined. We interpret this as comforting testimony to the linearity of the relations between expected returns and exposures to *HML* and *RMW*.

Armed with the evidence presented here, which version of the factors would we choose if starting fresh? We would put the Novy-Marx factors aside since the complications they introduce (sorts on industry demeaned variables and industry hedges) do not seem to produce asset pricing benefits. We would also dispense with the factors from the 2x3x3 sorts since they produce no asset pricing advantages relative to the factors from the simpler 2x2x2 sorts.

If we were starting from scratch, we might prefer the factors from the 2x2 *Size*-*B/M* and *Size*-*OP* sorts over the factors from the 2x3 sorts (the original approach). The attraction of the 2x2 sorts is that *HML* and *RMW* use all stocks and so are better diversified, whereas in the 2x3 sorts, 40% of the stocks are excluded in the construction of *HML* and *RMW*. In the tests of the four-factor model, however, the performance of these two sets of factors is much the same for the three sets of LHS portfolios examined here, so the choice between them seems inconsequential.

Judged on the *GRS* statistic and the average absolute intercept, the factors from the 2x2x2 sorts tend to perform a bit better than those from the 2x2 or 2x3 sorts. The *HML* and *RMW* slopes produced by the factors from the 2x2x2 sorts also better separate value and profitability exposures. This is, for example, an advantage for performance attribution in studies of portfolio performance. The 2x2x2 factors also have a downside. If we eventually wish to add factors to capture other patterns in average returns, the correlations among factor variables can result in poor diversification of some of the portfolios used to construct the factors. This is potentially a big disadvantage in tests of empirical asset pricing models.

For example, initiated by Jegadeesh and Titman (1993), there is a large literature documenting a momentum pattern in average returns, and it is now common to add a momentum factor to the three-factor model of Fama and French (1993). (See, for example, Carhart 1997 and Fama and French 2012.) Fama and French (2006) and Aharoni, Grundy, and Zeng (2013) find some evidence in favor of the relation between investment and average returns predicted by the valuation model of equation (3). If we use a 2x2x2x2x2 sort to construct *Size*, *B/M*, *OP*, investment, and momentum factors, some of the 32 building-block portfolios are likely to be poorly diversified. In contrast, when factors are constructed using separate 2x2 or 2x3 sorts on *Size* and each factor variable, the slopes may be harder to interpret, but all the building-block portfolios are likely to be well diversified.

We intend to pursue these issues in future drafts of this paper.

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Table 1 – Summary statistics for 25 *Size-B/M* portfolios; July 1963 to December 2012, 594 months

As in Fama and French (1993), at the end of each June stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Low to High), again using NYSE breakpoints. The intersections of the two sorts produce 25 value-weight *Size-B/M* portfolios. In the sort for June of year *t*, *B* is book equity at the end of the fiscal year ending in year *t*-1 and *M* is market cap at the end of December of year *t*-1, adjusted for changes in shares outstanding between the measurement of *B* and the end of December. The table shows averages and standard deviations of the monthly returns on the 25 portfolios in excess of the return on a one-month Treasury bill. The table also shows the average number of stocks and the average market cap (in millions of dollars) of the stocks in each portfolio, along with the average percent of the total sample of stocks and the average percent of aggregate market cap in the portfolio at the time of portfolio formation each year. The All columns show averages for all stocks in a *Size* group. The All rows show averages for all stocks in a *B/M* group. The sample, here and throughout, includes NYSE and AMEX stocks, with NASDAQ stocks added in 1973.

	Low *B/M*	2	3	4	High *B/M*	All	Low *B/M*	2	3	4	High *B/M*	All
	Average Monthly Excess Return						Standard Deviation of Monthly Excess Returns					
Small	0.19	0.76	0.80	0.97	1.11		8.08	6.96	6.07	5.72	6.17	
2	0.42	0.68	0.90	0.90	0.97		7.30	6.05	5.51	5.34	6.06	
3	0.45	0.73	0.75	0.85	1.03		6.73	5.55	5.08	4.96	5.54	
4	0.56	0.53	0.68	0.81	0.81		5.97	5.25	5.12	4.86	5.54	
Big	0.42	0.48	0.44	0.52	0.58		4.74	4.50	4.42	4.41	5.07	
	Average Number of Stocks						Average Market Cap					
Small	515.2	336.0	340.0	411.2	632.8	2,235.2	63	67	65	57	42	57
2	161.2	118.0	115.2	102.3	77.9	574.6	298	301	305	300	294	299
3	118.5	90.5	79.9	66.7	47.0	402.4	690	697	695	701	712	695
4	101.6	75.5	62.6	52.2	36.8	328.8	1,738	1,690	1,664	1,684	1,694	1699
Big	109.8	66.1	51.4	43.9	26.2	297.5	14,277	12,275	10,897	9,307	8,422	12,254
All	1,006.3	686.1	649.1	676.3	820.7	3,838.6	2,145	1,501	1,110	856	477	1,234
	Average Percent of Stocks						Average Percent of Total Market Cap					
Small	12.6	8.5	8.5	10.4	16.0	56.0	0.7	0.5	0.5	0.5	0.6	2.9
2	4.1	3.1	3.0	2.8	2.2	15.1	1.1	0.8	0.8	0.7	0.5	3.8
3	3.1	2.4	2.2	1.9	1.4	11.0	1.8	1.4	1.3	1.1	0.8	6.4
4	2.8	2.2	1.8	1.5	1.0	9.3	3.9	2.9	2.5	2.1	1.5	12.9
Big	3.2	2.0	1.5	1.3	0.7	8.6	32.6	15.8	11.7	9.0	4.8	74.0
All	25.7	18.1	17.1	17.8	21.3	100.0	40.1	21.4	16.7	13.5	8.2	100.0

Table 2 – Summary statistics for 25 *Size*-*OP* portfolios; July 1963 to December 2012, 594 months

At the end of each June stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five operating profitability (*OP*) groups (Low to High), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size*-*OP* portfolios. Operating profitability in the sort for June of year *t* is measured with accounting data for the fiscal year ending in year *t*-1 and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. The table shows averages and standard deviations of monthly returns on the 25 portfolios in excess of the one-month Treasury bill rate. The table also shows the average number of stocks and the average market cap of the stocks in each portfolio, along with the average percent of the total sample of stocks and the average percent of aggregate market cap in the portfolio at the time of portfolio formation each year. The All columns show averages for all stocks in a *Size* group. The All rows show averages for all stocks in an *OP* group.

	Low *OP*	2	3	4	High *OP*	All	Low *OP*	2	3	4	High *OP*	All
	Average Monthly Excess Return						Standard Deviation of Monthly Excess Returns					
Small	0.51	0.89	0.86	0.89	0.82		7.47	5.94	5.80	5.90	6.68	
2	0.54	0.74	0.79	0.77	0.93		7.11	5.75	5.45	5.59	6.36	
3	0.49	0.73	0.67	0.73	0.89		6.60	5.11	5.07	5.31	5.96	
4	0.51	0.61	0.58	0.66	0.78		6.05	5.01	4.96	5.00	5.52	
Big	0.33	0.28	0.39	0.43	0.54		5.32	4.50	4.44	4.53	4.40	
	Average Number of Stocks						Average Market Cap					
Small	1,067.6	340.1	303.2	260.6	313.6	2,285.1	47	61	66	71	70	56
2	139.6	103.2	111.0	107.3	110.2	571.4	286	299	305	310	306	299
3	73.5	72.2	81.9	87.7	84.8	400.2	675	696	698	706	707	695
4	49.8	60.4	67.5	74.4	73.2	325.2	1,642	1,703	1,661	1,726	1,750	1,699
Big	34.1	48.6	55.3	75.1	81.3	294.5	8,745	8,891	12,045	13,337	14,792	12,265
All	1,364.6	624.5	619.0	605.2	663.2	3,876.5	363	983	1,349	2,195	2,693	1,221
	Average Percent of Stocks						Average Percent of Total Market Cap					
Small	25.7	8.5	7.5	6.6	8.3	56.7	1.0	0.5	0.5	0.4	0.5	2.9
2	3.5	2.7	2.8	2.7	2.9	14.7	0.8	0.7	0.7	0.7	0.8	3.8
3	1.9	2.0	2.2	2.3	2.3	10.8	1.1	1.1	1.3	1.4	1.4	6.3
4	1.4	1.7	1.9	2.1	2.0	9.2	1.8	2.4	2.7	3.0	2.9	12.8
Big	1.0	1.5	1.7	2.2	2.3	8.7	5.4	10.2	13.7	19.1	25.8	74.1
All	33.4	16.4	16.2	16.0	17.9	100.0	10.1	14.9	18.8	24.6	31.5	100.0

Table 3 - Summary statistics for 32 portfolios formed on *Size*, *B/M*, and operating profitability, *OP*; July 1963 to December 2012, 594 months

At the end of June each year *t* stocks are allocated to two *Size* groups (Small and Big) using the NYSE median market cap as breakpoint. Stocks in each *Size* group are allocated independently to four *B/M* groups (Lo *B/M* to Hi *B/M* for fiscal year *t*-1) and four *OP* groups (Lo *OP* to Hi *OP* for fiscal year *t*-1) using NYSE breakpoints specific to the *Size* group. The table shows averages and standard deviations of the monthly returns (in excess of the one-month Treasury bill rate) on the 32 portfolios formed as the intersections of the three sorts. The table also shows the average number of stocks and the average market cap of the stocks in each portfolio, along with the percent of the total sample of stocks and the percent of aggregate market cap in the portfolio at the time of portfolio formation each year. The All columns show averages for all small or big stocks in a *OP* group. The All rows show averages for all small or big stocks in a *B/M* group.

	Small					Big				
	Lo *B/M*	2	3	Hi *B/M*	All	Lo *B/M*	2	3	Hi *B/M*	All
					Average of Monthly Excess Returns					
Lo *OP*	-0.04	0.69	0.80	0.88		0.18	0.19	0.32	0.55	
2	0.61	0.73	0.84	1.05		0.38	0.46	0.43	0.66	
3	0.61	0.84	1.03	1.26		0.36	0.55	0.64	0.85	
Hi *OP*	0.75	1.09	1.18	1.56		0.50	0.60	0.76	0.67	
					Standard Deviation of Monthly Excess Returns					
Lo *OP*	8.53	7.30	6.51	6.45		8.32	5.72	4.71	4.67	
2	7.23	5.60	5.06	5.81		6.08	4.90	4.45	4.65	
3	6.31	5.13	5.09	6.34		5.12	4.62	4.79	5.58	
Hi *OP*	6.25	5.64	5.84	7.91		4.65	4.85	5.70	7.07	
					Average Number of Stocks					
Lo *OP*	417.9	166.2	204.2	357.8	1,146.2	28.4	28.3	50.7	99.5	206.8
2	118.7	127.2	170.0	153.5	569.4	29.1	50.6	64.5	48.1	192.2
3	180.5	182.4	150.3	79.8	593.1	61.7	72.4	47.3	17.9	199.3
Hi *OP*	402.2	146.5	80.1	42.4	671.1	124.6	45.0	21.1	8.3	199.0
All	1,119.3	622.3	604.6	633.5	2,979.7	243.7	196.3	183.6	173.7	797.3
					Average Market Cap					
Lo *OP*	101	101	92	68	89	3,339	3,546	3,909	3,595	3,606
2	190	184	150	107	153	5,716	4,666	4,484	3,691	4,584
3	223	196	146	104	178	7,412	5,804	5,335	6,082	6,249
Hi *OP*	218	175	149	96	190	8,156	6,376	4,830	4,888	7,328
All	167	161	128	84	138	6,890	5,301	4,586	3,911	5,373
					Average Percent of Stocks					
Lo *OP*	9.8	4.1	5.1	9.3	28.4	0.7	0.8	1.5	3.0	6.0
2	3.1	3.4	4.5	4.0	15.0	0.8	1.5	1.9	1.4	5.7
3	4.7	4.8	3.9	2.0	15.4	1.7	2.2	1.4	0.5	5.8
Hi *OP*	11.0	3.9	2.0	1.0	17.9	3.7	1.3	0.6	0.2	5.9
All	28.6	16.2	15.6	16.3	76.7	6.9	5.8	5.4	5.1	23.3
					Average Percent of Total Market Cap					
Lo *OP*	0.8	0.3	0.4	0.6	2.1	1.7	2.4	4.6	7.4	16.0
2	0.5	0.5	0.6	0.4	1.0	3.4	5.4	6.8	4.5	20.1
3	0.8	0.8	0.5	0.2	2.4	8.6	8.1	5.0	2.0	23.7
Hi *OP*	1.9	0.6	0.3	0.1	2.9	21.6	6.0	2.3	0.9	30.8
All	4.0	2.2	1.8	1.3	9.3	35.4	21.8	18.7	14.8	90.7

Table 4 – Construction of *Size*, *B/M*, and profitability factors

We use independent sorts to assign stocks to two *Size* groups, one, two, or three *B/M* groups, and one, two, or three operating profitability (*OP*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label the portfolios with three letters. The first describes the *Size* group, small (*S*) or big (*B*). The second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), or the *OP* group, robust (*R*), neutral (*N*), or weak (*W*) if we do not sort on B/M. The third character in the sorts on all three variables is the profitability group. The factors are *SMB* (small minus big), *HML* (high *B/M* minus low *B/M*), and *RMW* (robust operating profitability minus weak operating profitability). The subscripts on the factors identify the sorts used to construct them.

Sort	Breakpoints	Factors and their components
2x3 sorts on *Size* and *B/M* or *Size* and *OP*	*Size*: NYSE median	$SMB_{23,B/M} = (SH + SN + SL) / 3 - (BH + BN + BL) / 3$
		$SMB_{23,OP} = (SR + SN + SW) / 3 - (BR + BN + BW) / 3$
		$SMB_{23} = (SMB_{23,B/M} + SMB_{23,OP}) / 2$
	B/M: 30th & 70th NYSE percentiles	$HML_{23} = (SH + BH) / 2 - (SL + BL) / 2$
	OP: 30th & 70th NYSE percentiles	$RMW_{23} = (SR + BR) / 2 - (SW + BW) / 2$
2x2 sorts on *Size* and *B/M* or *Size* and *OP*	*Size*: NYSE median	$SMB_{22} = (SH + SL + SR + SW) / 4 - (BH + BL + BR + BW) / 4$
	B/M: NYSE median	$HML_{22} = (SH + BH) / 2 - (SL + BL) / 2$
	OP: NYSE median	$RMW_{22} = (SR + BR) / 2 - (SW + BW) / 2$
2x2x2 sorts on *Size*, *B/M*, and *OP*	*Size*: NYSE median	$SMB_{222} = (SHR + SHW + SLR + SLW) / 4 - (BHR + BHW + BLR + BLW) / 4$
	B/M: NYSE median	$HML_{222} = (SHR + SHW + BHR + BHW) / 4 - (SLR + SLW + BLR + BLW) / 4$
	OP: NYSE median	$RMW_{222} = (SHR + SLR + BHR + BLR) / 4 - (SHW + SLW + BHW + BLW) / 4$
2x3x3 sorts on *Size*, *B/M*, and *OP*	*Size*: NYSE median	$SMB_{233} = (SHR + SHN + SHW + SNR + SNN + SNW + SLR + SLN + SLW) / 9 - (BHR + BHN + BHW + BNR + BNN + BNW + BLR + BLN + BLW) / 9$
	B/M: 30th & 70th NYSE percentiles	$HML_{233} = (SHR + SHN + SHW + BHR + BHN + BHW) / 6 - (SLR + SLN + SLW + BLR + BLN + BLW) / 6$
	OP: 30th & 70th NYSE percentiles	$RMW_{233} = (SHR + SNR + SLR + BHR + BNR + BLR) / 6 - (SHW + SNW + SLW + BHW + BNW + BLW) / 6$

Table 5 – Summary statistics for monthly factor returns; July 1963 to December 2012, 594 months

Mkt is R_M - R_F, the value-weight return on the market portfolio of all sample stocks, minus the riskfree rate (the one month Treasury bill rate). At the end of June each year, stocks are assigned to two *Size* groups using the NYSE median market cap as breakpoint. Stocks are also assigned independently to one, two, or three book-to-market equity (*B/M*) groups and one, two, or three operating profitability (*OP*) groups, using NYSE medians of *B/M* and *OP* or the 30th and 70th NYSE percentiles. In the first two blocks of Part A, the *B/M* factor, *HML*, uses the VW portfolios formed from the intersection of the *Size* and *B/M* sorts (2x2 = 4 or 2x3 = 6 portfolios), and the profitability factor, *RMW*, uses four or six VW portfolios formed from the intersection of the *Size* and *OP* sorts. In the third and fourth blocks, *HML* and *RMW* use the intersections of the *Size*, *B/M*, and *OP* sorts (2x2x2 = 8 or 2x3x3 = 18 portfolios). HML_B is the average return on the portfolio(s) of big high *B/M* stocks minus the average return on the portfolio(s) of big low *B/M* stocks, HML_S is the same but for portfolios of small stocks, *HML* is the average of HML_S and HML_B, and HML_{S-B} is the difference between them. RMW_S, RMW_B, *RMW*, and RMW_{S-B} are defined in the same way, but use high and low *OP* instead of *B/M*. In the 2x2x2 and 2x3x3 sorts, *SMB* is the average return on the four or nine portfolios of small stocks minus the average return on the four or nine portfolios of big stocks. In the separate 2x3 *Size-B/M* and *Size-OP* sorts, there are two versions of *SMB*, $SMB_{23,B/M}$ and $SMB_{23,OP}$ and *SMB* is the average of the two. Similarly, *SMB* in the separate 2x2 sorts is the average of $SMB_{22,B/M}$ and $SMB_{22,OP}$. The 2x3 Novy-Marx *HML* and *RMW* factors are defined in the same way as the factors from the 2x3 sorts in the first block of Part A, except that the *B/M* and profitability sorts use industry demeaned version of the variables and the positions in individual stocks are balanced by offsetting positions in their industry portfolios. Part A of the table shows average monthly returns (Ave), the standard deviations of monthly returns (Std Dev) and the *t*-statistics for the average returns (*t*-stat). Part B shows correlation matrices for *Mkt*, *SMB*, *HML*, and *RMW*.

Part A: Averages, standard deviations, and *t*-statistics for the average monthly returns

	Mkt	*SMB*	*HML*	HML_S	HML_B	HML_{S-B}	*RMW*	RMW_S	RMW_B	RMW_{S-B}
2x3 *Size-B/M* and *Size-OP* factors										
Ave	0.46	0.29	0.38	0.55	0.21	0.34	0.26	0.33	0.19	0.14
Std Dev	4.51	3.06	2.90	3.27	3.13	2.71	2.15	2.71	2.36	2.70
t-stat	2.47	2.29	3.22	4.10	1.67	3.02	2.93	2.97	1.94	1.29
2x2 *Size-B/M* and *Size-OP* factors										
Ave	0.46	0.29	0.29	0.41	0.16	0.25	0.17	0.21	0.14	0.08
Std Dev	4.51	3.11	2.18	2.41	2.38	1.98	1.53	1.94	1.70	2.00
t-stat	2.47	2.29	3.23	4.20	1.66	3.11	2.77	2.65	1.94	0.92
2x2x2 *Size-B/M-OP* factors										
Ave	0.46	0.28	0.36	0.46	0.25	0.22	0.27	0.31	0.22	0.10
Std Dev	4.51	2.98	2.34	2.61	2.51	2.06	1.52	2.10	1.57	2.12
t-stat	2.47	2.30	3.71	4.35	2.40	2.56	4.24	3.63	3.37	1.10
2x3x3 *Size-B/M-OP* factors										
Ave	0.46	0.31	0.77	1.03	0.57	0.46	0.62	0.76	0.46	0.30
Std Dev	4.51	2.91	4.75	5.14	5.39	4.34	3.40	4.29	3.60	4.34
t-stat	2.47	2.59	3.94	4.90	2.60	2.59	4.47	4.32	3.12	1.68
2x3 Novy-Marx industry-adjusted industry-hedged factors										
Ave	0.46	0.29	0.34				0.25			
Std Dev	4.51	3.06	1.28				1.15			
t-stat	2.47	2.29	6.56				5.36			

Table 5, Part B: Correlations

	Mkt	*SMB*	*HML*	*RMW*
2x3 *Size-B/M* and 2x3 *Size-OP* factors				
Mkt	1.00	0.29	-0.30	-0.21
SMB_{23}	0.29	1.00	-0.14	-0.32
HML_{23}	-0.30	-0.14	1.00	0.08
RMW_{23}	-0.21	-0.32	0.08	1.00
2x2 *Size-B/M* and 2x2 *Size-OP* factors				
Mkt	1.00	0.29	-0.35	-0.12
SMB_{22}	0.29	1.00	-0.18	-0.29
HML_{22}	-0.35	-0.18	1.00	0.04
RMW_{22}	-0.12	-0.29	0.04	1.00
2x2x2 *Size-B/M-OP* factors				
Mkt	1.00	0.27	-0.37	-0.24
SMB_{222}	0.27	1.00	-0.22	-0.30
HML_{222}	-0.37	-0.22	1.00	0.59
RMW_{222}	-0.24	-0.30	0.59	1.00
2x3x3 *Size-B/M-OP* factors				
Mkt	1.00	0.23	-0.30	-0.26
SMB_{233}	0.23	1.00	-0.16	-0.29
HML_{233}	-0.30	-0.16	1.00	0.66
RMW_{233}	-0.26	-0.29	0.66	1.00
2x3 Novy-Marx industry-adjusted industry-hedged factors				
Mkt	1.00	0.29	-0.22	-0.30
SMB_{23}	0.29	1.00	-0.02	-0.27
HML_{NM}	-0.22	-0.02	1.00	-0.08
RMW_{NM}	-0.30	-0.27	-0.08	1.00

Table 6 – Summary statistics for tests of three- and four-factor models; July 1963 to December 2012

The table tests the ability of three- and four-factor models to explain monthly excess returns on 25 *Size-B/M* portfolios (Part A), 25 *Size-OP* portfolios (Part B), and 32 *Size-B/M-OP* portfolios (Part C). For each set of 25 or 32 regressions, the table shows the factors used as explanatory variables (Model), the *GRS* statistic of Gibbons, Ross, and Shanken (1989) testing whether the expected values of all 25 or 32 intercepts are zero, the *p*-value of the *GRS* statistic, the average absolute value of the intercepts (Ave |*a*|), the average of the regression R^2 (Ave R^2), and the average standard error of the intercepts (SE(*a*)).

Part A: Summary of three- and four-factor regression intercepts for 25 *Size-B/M* portfolios

Model	*GRS*	*p*-value	Ave |*a*|	Ave R^2	SE(*a*)
2x3 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{23} HML_{23}	3.60	1.000	0.101	0.92	0.07
Mkt SMB_{23} HML_{23} RMW_{23}	3.12	1.000	0.097	0.92	0.07
2x2 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{22} HML_{22}	3.55	1.000	0.100	0.92	0.07
Mkt SMB_{22} HML_{22} RMW_{22}	3.13	1.000	0.097	0.92	0.07
2x2x2 *Size-B/M-OP* factors					
Mkt SMB_{222} HML_{222}	3.32	1.000	0.096	0.91	0.07
Mkt SMB_{222} HML_{222} RMW_{222}	2.99	1.000	0.091	0.91	0.07
2x3x3 *Size-B/M-OP* factors					
Mkt SMB_{233} HML_{233}	3.36	1.000	0.104	0.90	0.07
Mkt SMB_{233} HML_{233} RMW_{233}	2.99	1.000	0.091	0.91	0.07
2x3 Novy-Marx *Size-B/M Size-OP* factors					
Mkt SMB_{23} HML_{NM}	3.01	1.000	0.095	0.89	0.08
Mkt SMB_{23} HML_{NM} RMW_{NM}	2.42	1.000	0.089	0.89	0.08

Table 6, Part B: Summary of three- and four-factor regression intercepts for 25 *Size-OP* portfolios

Model	*GRS*	*p*-value	Ave $\|a\|$	Ave R^2	SE(a)
2x3 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{23} RMW_{23}	1.62	0.970	0.072	0.92	0.07
Mkt SMB_{23} HML_{23} RMW_{23}	1.65	0.974	0.063	0.93	0.06
2x2 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{22} RMW_{22}	1.72	0.983	0.079	0.92	0.07
Mkt SMB_{22} HML_{22} RMW_{22}	1.72	0.983	0.059	0.93	0.06
2x2x2 *Size-B/M-OP* factors					
Mkt SMB_{222} RMW_{222}	1.64	0.974	0.061	0.91	0.07
Mkt SMB_{222} HML_{222} RMW_{222}	1.50	0.942	0.063	0.92	0.07
2x3x3 *Size-B/M-OP* factors					
Mkt SMB_{233} RMW_{233}	1.69	0.980	0.064	0.91	0.07
Mkt SMB_{233} HML_{233} RMW_{233}	1.64	0.973	0.074	0.91	0.07
2x3 Novy-Marx *Size-B/M Size-OP* factors					
Mkt SMB_{NM} RMW_{NM}	1.74	0.985	0.078	0.91	0.07
Mkt SMB_{23} HML_{NM} RMW_{NM}	1.64	0.974	0.096	0.91	0.07

Part C: Summary of three- and four-factor regression intercepts for 32 *Size-B/M-OP* portfolios

Model	*GRS*	*p*-value	Ave $\|a\|$	Ave R^2	SE(a)
2x3 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{23} HML_{23} RMW_{23}	1.98	0.999	0.112	0.85	0.09
2x2 *Size-B/M* and *Size-OP* factors					
Mkt SMB_{22} HML_{22} RMW_{22}	2.30	1.000	0.113	0.85	0.09
2x2x2 *Size-B/M-OP* factors					
Mkt SMB_{222} HML_{222} RMW_{222}	1.93	0.998	0.103	0.85	0.09
2x3x3 *Size-B/M-OP* factors					
Mkt SMB_{233} HML_{233} RMW_{233}	1.92	0.998	0.102	0.85	0.09
2x3 Novy-Marx *Size-B/M Size-OP* factors					
Mkt SMB_{23} HML_{NM} RMW_{NM}	1.54	0.970	0.118	0.82	0.11

Table 7 – Regression results for 25 *Size*-*B/M* portfolios; July 1963 to December 2012, 594 months

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Lo *B/M* to Hi *B/M*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size*-*B/M* portfolios. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size*-*B/M* portfolios. The RHS variables are the excess market return, $Mkt = R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML*, and the profitability factor, *RMW*, constructed using either independent 2x3 sorts on *Size* and *B/M* and *Size* and *OP* or the Novy-Marx (2012) factors. Part A of the table shows three-factor and four-factor intercepts. Part B shows four-factor regression slopes for *HML* and *RMW*.

Part A: Three-factor and four-factor regression intercepts

			a					*t*(*a*)		
	Lo *B/M*	2	3	4	Hi *B/M*	Lo *B/M*	2	3	4	Hi *B/M*
Standard 2x3 factors: Mkt SMB_{23} HML_{23}										
Small	-0.52	-0.01	0.00	0.13	0.12	-5.33	-0.16	0.01	2.42	1.99
2	-0.19	-0.05	0.11	0.06	-0.04	-3.02	-0.96	2.00	1.13	-0.76
3	-0.06	0.04	0.01	0.06	0.11	-0.99	0.61	0.20	0.89	1.45
4	0.14	-0.11	-0.04	0.06	-0.09	2.21	-1.51	-0.55	0.95	-1.10
Big	0.17	0.03	-0.07	-0.11	-0.18	3.52	0.53	-1.00	-1.83	-1.91
Standard 2x3 factors: Mkt SMB_{23} HML_{23} RMW_{23}										
Small	-0.34	0.12	0.03	0.15	0.14	-3.86	1.84	0.60	2.61	2.38
2	-0.14	-0.08	0.05	0.03	-0.05	-2.28	-1.44	1.01	0.49	-0.77
3	-0.02	-0.01	-0.06	0.02	0.06	-0.39	-0.21	-0.91	0.24	0.84
4	0.17	-0.17	-0.10	0.07	-0.11	2.74	-2.44	-1.32	0.96	-1.38
Big	0.10	-0.04	-0.07	-0.15	-0.14	2.11	-0.64	-0.98	-2.40	-1.42
2x2x2 *Size*-*B/M*-*OP* factors: Mkt SMB_{222} HML_{222}										
Small	-0.46	0.01	-0.02	0.09	0.05	-4.79	0.11	-0.39	1.50	0.85
2	-0.14	-0.06	0.06	-0.01	-0.11	-2.14	-1.04	1.04	-0.11	-1.69
3	-0.01	0.02	-0.04	-0.01	0.04	-0.19	0.31	-0.60	-0.12	0.52
4	0.19	-0.12	-0.09	0.01	-0.15	2.82	-1.65	-1.24	0.11	-1.72
Big	0.20	0.02	-0.10	-0.19	-0.20	3.71	0.33	-1.44	-3.23	-1.94
2x2x2 *Size*-*B/M*-*OP* factors: Mkt SMB_{222} HML_{222} RMW_{222}										
Small	-0.37	0.10	0.02	0.12	0.10	-3.99	1.37	0.30	2.18	1.61
2	-0.13	-0.06	0.05	0.01	-0.07	-1.96	-1.05	0.90	0.21	-1.06
3	-0.02	0.00	-0.05	0.02	0.07	-0.31	0.02	-0.72	0.30	0.89
4	0.18	-0.13	-0.08	0.06	-0.11	2.70	-1.74	-1.14	0.95	-1.29
Big	0.13	-0.02	-0.07	-0.16	-0.12	2.64	-0.30	-1.05	-2.82	-1.15
2x3 Novy-Marx factors: Mkt SMB_{23} HML_{NM}										
Small	-0.40	-0.00	-0.02	0.12	0.07	-3.92	-0.04	-0.32	1.78	0.92
2	-0.07	-0.08	0.07	0.02	-0.12	-1.02	-1.48	1.18	0.32	-1.39
3	0.01	0.02	-0.04	0.01	0.02	0.07	0.28	-0.60	0.11	0.19
4	0.17	-0.16	-0.12	0.01	-0.18	2.26	-2.12	-1.46	0.12	-1.74
Big	0.18	0.02	-0.12	-0.15	-0.19	3.09	0.35	-1.49	-1.78	-1.54
2x3 Novy-Marx factors: Mkt SMB_{23} HML_{NM} RMW_{NM}										
Small	-0.16	0.17	0.03	0.16	0.21	-1.58	2.27	0.51	2.41	2.62
2	-0.07	-0.16	0.04	0.02	-0.03	-1.00	-2.68	0.57	0.22	-0.34
3	-0.02	-0.05	-0.08	0.02	-0.02	-0.24	-0.67	-1.07	0.20	-0.16
4	0.12	-0.26	-0.13	0.05	-0.13	1.54	-3.34	-1.56	0.58	-1.22
Big	0.05	-0.06	-0.06	-0.12	-0.01	0.85	-0.86	-0.72	-1.37	-0.09

Table 7, Part B: Four-factor regression slopes

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + e(t)$$

	Lo *B/M*	2	3	4	Hi *B/M*	Lo *B/M*	2	3	4	Hi *B/M*
Standard 2x3 factors: *Mkt* SMB_{23} HML_{23} RMW_{23}										
			h					*t*(*h*)		
Small	-0.45	-0.09	0.17	0.35	0.59	-14.54	-3.94	8.99	17.75	27.59
2	-0.48	0.05	0.32	0.50	0.72	-21.62	2.44	17.49	26.32	34.84
3	-0.51	0.14	0.41	0.58	0.72	-23.07	5.88	17.99	25.62	27.68
4	-0.46	0.20	0.44	0.56	0.79	-20.75	7.86	17.39	23.01	27.16
Big	-0.34	0.13	0.32	0.62	0.77	-21.24	6.14	12.54	28.55	23.21
			r					*t*(*r*)		
Small	-0.52	-0.39	-0.10	-0.04	-0.07	-12.46	-12.34	-3.62	-1.33	-2.50
2	-0.14	0.08	0.16	0.10	0.00	-4.61	2.96	6.24	3.92	0.14
3	-0.11	0.16	0.21	0.12	0.13	-3.62	5.06	6.81	3.99	3.69
4	-0.10	0.19	0.16	-0.01	0.07	-3.28	5.58	4.69	-0.17	1.74
Big	0.22	0.20	-0.00	0.10	-0.13	9.99	7.24	-0.07	3.47	-2.87
2x2x2 *Size-B/M-OP* factors: *Mkt* SMB_{222} HML_{222} RMW_{222}										
			h					*t*(*h*)		
Small	-0.41	0.04	0.30	0.53	0.84	-8.27	1.16	10.00	17.90	24.99
2	-0.58	0.06	0.41	0.69	0.97	-16.14	1.88	14.17	23.57	28.26
3	-0.65	0.14	0.52	0.79	0.95	-18.27	3.69	14.42	22.79	22.58
4	-0.60	0.22	0.59	0.83	1.02	-16.98	5.52	15.00	22.78	21.91
Big	-0.56	0.08	0.47	0.87	1.05	-21.62	2.39	12.56	28.23	19.28
			r					*t*(*r*)		
Small	-0.57	-0.55	-0.25	-0.23	-0.29	-7.75	-9.71	-5.42	-5.14	-5.80
2	-0.07	0.01	0.05	-0.11	-0.26	-1.25	0.16	1.05	-2.46	-5.03
3	0.05	0.13	0.05	-0.17	-0.18	0.89	2.28	0.97	-3.23	-2.88
4	0.04	0.05	-0.04	-0.35	-0.23	0.81	0.82	-0.72	-6.46	-3.32
Big	0.42	0.24	-0.17	-0.15	-0.54	10.73	4.93	-3.02	-3.18	-6.54
2x3 Novy-Marx factors: *Mkt* SMB_{23} HML_{NM} RMW_{NM}										
			h					*t*(*h*)		
Small	-1.00	-0.21	0.26	0.49	0.85	-13.19	-3.74	5.84	9.81	14.00
2	-0.96	0.17	0.52	0.76	1.11	-17.74	4.02	11.25	13.98	17.13
3	-0.84	0.26	0.70	0.88	1.21	-14.36	4.92	12.23	13.88	16.28
4	-0.68	0.43	0.80	0.86	1.27	-11.46	7.51	12.73	13.30	15.56
Big	-0.43	0.22	0.52	0.91	0.97	-9.76	4.62	8.63	13.87	10.47
			r					*t*(*r*)		
Small	-0.69	-0.49	-0.14	-0.14	-0.40	-7.85	-7.72	-2.74	-2.37	-5.74
2	0.01	0.21	0.10	0.02	-0.25	0.12	4.16	1.89	0.30	-3.31
3	0.07	0.19	0.11	-0.02	0.10	1.03	3.13	1.66	-0.34	1.16
4	0.14	0.28	0.04	-0.12	-0.14	2.08	4.31	0.58	-1.56	-1.45
Big	0.38	0.22	-0.16	-0.08	-0.50	7.43	3.99	-2.36	-1.11	-4.68

Table 8 – Four-factor regressions for 5x5 *Size-OP* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *OP* groups (Lo *OP* to Hi *OP*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-OP* portfolios. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-OP* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML*, and the profitability factor, *RMW*, constructed using either independent 2x3 sorts on *Size* and *B/M* and *Size* and *OP* or 2x2x2 sorts on *Size*, *B/M*, and *OP*. Part A of the table shows four-factor intercepts; Part B shows four-factor regression slopes for *HML* and *RMW*.

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + e(t)$$

	Lo *OP*	2	3	4	Hi *OP*	Lo *OP*	2	3	4	Hi *OP*
Part A: Regression Intercepts										
			a					*t*(*a*)		
2x3 *Size-B/M* and *Size-OP* factors										
Small	-0.07	0.03	-0.03	-0.06	-0.18	-0.96	0.46	-0.47	-0.93	-2.57
2	-0.04	-0.07	-0.03	-0.10	-0.04	-0.65	-1.21	-0.53	-1.80	-0.58
3	0.04	0.08	-0.05	-0.07	0.00	0.61	1.19	-0.94	-1.13	0.05
4	0.14	0.06	-0.10	-0.05	0.06	1.74	0.90	-1.62	-0.76	0.82
Big	0.06	-0.10	0.01	0.01	0.09	0.89	-1.59	0.23	0.16	2.04
2x2x2 *Size-B/M-OP* factors										
Small	-0.12	0.02	-0.03	-0.06	-0.18	-1.45	0.38	-0.53	-0.96	-2.41
2	-0.07	-0.05	-0.03	-0.11	-0.01	-1.04	-0.77	-0.47	-1.73	-0.17
3	0.00	0.10	-0.04	-0.07	0.04	0.02	1.47	-0.62	-1.13	0.49
4	0.10	0.08	-0.08	-0.02	0.07	1.16	1.16	-1.31	-0.32	1.04
Big	0.03	-0.09	0.02	-0.01	0.12	0.33	-1.55	0.32	-0.30	2.47
Part B: Regression Slopes										
2x2x2 *Size-B/M-OP* factors: *Mkt* SMB_{222} HML_{222} RMW_{222}										
			h					*t*(*h*)		
Small	-0.00	0.15	0.17	0.10	-0.08	-0.02	4.35	5.32	2.67	-2.04
2	0.02	0.19	0.14	0.01	-0.18	0.47	5.60	4.59	0.32	-4.63
3	0.16	0.24	0.19	0.09	-0.24	3.54	6.64	5.73	2.73	-6.02
4	0.29	0.41	0.27	0.10	-0.19	6.29	11.38	7.89	2.78	-4.91
Big	0.42	0.48	0.31	-0.11	-0.33	10.08	14.81	10.19	-4.57	-12.88
			r					*t*(*r*)		
Small	-0.94	0.13	0.27	0.54	0.63	-14.12	2.54	5.40	9.74	10.18
2	-0.88	0.03	0.28	0.50	0.76	-16.15	0.50	5.98	9.32	12.76
3	-1.08	-0.21	0.17	0.43	0.82	-15.90	-3.74	3.35	8.23	13.14
4	-1.15	-0.56	0.09	0.38	0.56	-16.10	-10.06	1.70	7.21	9.68
Big	-1.04	-0.71	-0.42	0.34	0.64	-16.07	-14.25	-8.99	9.56	16.32

Table 9 – Four-factor regression results for 32 *Size-B/M-OP* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *B/M* groups (Lo *B/M* to Hi *B/M*) and four *OP* groups (Lo *OP* to Hi *OP*), using NYSE *B/M* and *OP* breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-B/M-OP* portfolios. The LHS variables in each set of 32 regressions are the excess returns on the 32 *Size-B/M-OP* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the *B/M* factor, *HML*, and the profitability factor, *RMW*, constructed using either independent 2x3 sorts on *Size* and *B/M* and *Size* and *OP* or 2x2x2 sorts on *Size*, *B/M* and *OP*. Part A of the table shows four-factor intercepts. Part B shows four-factor regression slopes for *HML* and *RMW*.

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + e(t)$$

	Small								Big							
	Lo *B/M*	2	3	Hi *B/M*	Lo *B/M*	2	3	Hi *B/M*	Lo *B/M*	2	3	Hi *B/M*	Lo *B/M*	2	3	Hi *B/M*
Part A: Intercepts																
	a				*t*(*a*)				*a*				*t*(*a*)			
Standard 2x3 factors																
Lo *OP*	-0.38	0.08	0.00	-0.09	-4.06	0.87	0.06	-1.35	0.14	-0.18	-0.11	-0.11	0.80	-1.71	-1.34	-1.86
2	-0.02	-0.09	-0.00	0.01	-0.19	-1.31	-0.08	0.18	0.23	-0.08	-0.10	-0.15	2.05	-0.90	-1.25	-1.91
3	-0.13	-0.02	0.10	0.21	-2.08	-0.42	1.83	1.77	0.02	-0.01	-0.08	0.07	0.28	-0.08	-0.85	0.54
Hi *OP*	-0.16	0.06	0.12	0.35	-3.34	1.09	1.28	1.80	0.07	-0.05	0.04	-0.32	1.29	-0.50	0.29	-1.58
2x2x2 *Size-B/M-OP* factors																
Lo *OP*	-0.41	0.07	0.01	-0.13	-4.02	0.69	0.17	-1.76	0.15	-0.16	-0.09	-0.09	0.84	-1.51	-1.14	-1.49
2	0.03	-0.06	0.00	-0.02	0.28	-0.73	0.06	-0.21	0.25	-0.00	-0.08	-0.14	2.29	-0.01	-1.02	-1.69
3	-0.10	-0.01	0.07	0.18	-1.48	-0.17	1.23	1.47	0.04	0.01	-0.11	-0.01	0.65	0.21	-1.26	-0.06
Hi *OP*	-0.12	0.04	0.07	0.28	-2.02	0.62	0.72	1.47	0.12	-0.04	0.01	-0.41	1.96	-0.37	0.06	-2.08
Part B: *HML* and *RMW* slopes, 2x2x2 *Size-B/M-OP* factors																
	h				*t*(*h*)				*h*				*t*(*h*)			
Lo *OP*	-0.41	0.17	0.55	0.92	-7.52	3.26	13.82	23.83	-0.94	0.01	0.49	0.95	-10.12	0.16	11.73	29.20
2	-0.39	0.32	0.68	1.05	-7.65	7.86	22.34	26.54	-0.70	0.06	0.56	1.05	-11.80	1.20	13.21	24.42
3	-0.23	0.43	0.75	0.97	-6.58	13.79	25.70	14.99	-0.67	0.08	0.62	0.66	-18.73	2.33	13.54	9.79
Hi *OP*	-0.19	0.54	0.79	1.01	-6.15	16.91	16.31	9.87	-0.50	0.10	0.39	0.89	-15.13	1.87	5.51	8.48
	r				*t*(*r*)				*r*				*t*(*r*)			
Lo *OP*	-1.22	-0.90	-0.69	-0.46	-14.95	-11.82	-11.38	-7.91	-0.70	-0.54	-0.66	-0.61	-5.04	-6.61	-10.55	-12.47
2	-0.18	-0.00	-0.15	-0.12	-2.30	-0.04	-3.32	-2.03	-0.26	0.05	-0.33	-0.17	-2.94	0.75	-5.15	-2.70
3	0.20	0.21	0.16	-0.04	3.84	4.38	3.54	-0.46	0.58	0.29	0.28	0.39	10.77	5.35	4.09	3.89
Hi *OP*	0.70	0.52	0.43	0.21	15.04	10.66	5.87	1.39	0.61	0.60	0.38	0.54	12.07	7.75	3.55	3.40

Table A1 – Summary statistics for portfolios used to construct *SMB*, *HML*, and *RMW*; July 1963 - December 2012, 594 months

We use independent sorts to assign stocks to two *Size* groups, one, two, or three *B/M* groups, and one, two, or three operating profitability (*OP*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label the portfolios with three letters. The first describes the *Size* group, small (*S*) or big (*B*). The second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), or the *OP* group, robust (*R*), neutral (*N*), or weak (*W*) if we do not sort on B/M. The third character in the sorts on all three variables is the profitability group.

2x3 *Size*-*B/M* portfolios

	SL	*SN*	*SH*	*BL*	*BN*	*BH*
Ave	0.88	1.28	1.43	0.85	0.91	1.07
Std Dev	6.92	5.47	5.62	4.68	4.37	4.70
t-stat	3.11	5.69	6.21	4.45	5.10	5.54

2x3 *Size*-*OP* portfolios

	SW	*SN*	*SR*	*BW*	*BN*	*BR*
Ave	0.98	1.23	1.31	0.77	0.84	0.95
Std Dev	6.70	5.35	6.00	5.00	4.40	4.42
t-stat	3.58	5.62	5.34	3.74	4.65	5.26

2x2 *Size*-*B/M* portfolios

	SL	*SH*	*BL*	*BH*
Ave	0.99	1.40	0.85	1.01
Std Dev	6.45	5.45	4.53	4.40
t-stat	3.73	6.27	4.57	5.60

2x2 *Size*-*OP* portfolios

	SW	*SR*	*BW*	*BR*
Ave	1.07	1.28	0.79	0.92
Std Dev	6.20	5.72	4.55	4.42
t-stat	4.20	5.45	4.21	5.08

2x2x2 *Size*-*B/M*-*OP* portfolios

	SLW	*SLR*	*SHW*	*SHR*	*BLW*	*BLR*	*BHW*	*BHR*
Ave	0.79	1.17	1.32	1.57	0.72	0.90	0.93	1.18
Std Dev	7.28	5.95	5.55	5.32	5.20	4.48	4.37	4.89
t-stat	2.66	4.78	5.79	7.21	3.37	4.88	5.18	5.91

2x3x3 *Size*-*B/M*-*OP* portfolios

	SLW	*SLN*	*SLR*	*SNW*	*SNN*	*SNR*	*SHW*	*SHN*	*SHR*
Ave	0.45	0.98	1.16	1.16	1.24	1.48	1.32	1.53	1.81
Std Dev	8.39	6.62	6.30	6.51	5.04	5.63	5.92	5.23	6.37
t-stat	1.32	3.60	4.49	4.34	6.01	6.41	5.44	7.13	6.93

	BLW	*BLN*	*BLR*	*BNW*	*BNN*	*BNR*	*BHW*	*BHN*	*BHR*
Ave	0.52	0.77	0.92	0.82	0.91	1.06	0.97	1.12	1.26
Std Dev	7.14	5.06	4.51	4.75	4.34	4.87	4.87	4.87	6.56
t-stat	1.78	3.69	4.98	4.21	5.09	5.30	4.87	5.62	4.69